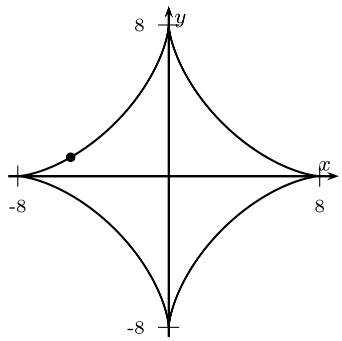
Use implicit differentiation to find an equation of the tangent line to the curve at the given point.[1]

$$x^{2/3} + y^{2/3} = 4, \quad \left(-3\sqrt{3}, 1\right), \quad (astroid)$$



To write the equation of the tangent line at the given point, we need the slope,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{2/3} + y^{2/3}\right] &= \frac{\mathrm{d}}{\mathrm{d}x}[4] \\
\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{2/3}\right] + \frac{\mathrm{d}}{\mathrm{d}x}\left[y^{2/3}\right] &= \frac{\mathrm{d}}{\mathrm{d}x}[4] \\
\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} &= 0 \\
\frac{3}{2}\left(\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{\mathrm{d}y}{\mathrm{d}x}\right) &= \frac{3}{2} \cdot 0 \\
x^{-1/3} + y^{-1/3}\frac{\mathrm{d}y}{\mathrm{d}x} &= 0 \\
y^{-1/3}\frac{\mathrm{d}y}{\mathrm{d}x} &= -x^{-1/3} \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{-x^{-1/3}}{y^{-1/3}} \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{y^{1/3}}{x^{1/3}}
\end{aligned}$$

Substituting the given values, $x = -3\sqrt{3}$ and $y = 1$, we get

$$\begin{aligned}
\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right]_{(-3\sqrt{3},1)} &= -\frac{(1)^{1/3}}{\left(-3\sqrt{3}\right)^{1/3}} \\
&= -\frac{1}{\left(-1 \cdot 3 \cdot 3^{1/2}\right)^{1/3}} \\
&= -\frac{1}{(-1)^{1/3} \cdot (3)^{1/3} \cdot \left(3^{1/2}\right)^{1/3}} \\
&= -\frac{1}{-1 \cdot (3)^{1/3} \cdot 3^{1/6}} \\
&= \frac{1}{3^{1/2}} \\
&= \frac{1}{\sqrt{3}}
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 215, #30.

Using point-slope form for the equation of a line, we get

$$\begin{aligned} y - 1 &= \frac{1}{\sqrt{3}}\left(x - \left(-3\sqrt{3}\right)\right) \\ y &= \frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}\left(3\sqrt{3}\right) + 1 \\ y &= \frac{1}{\sqrt{3}}x + 4 \end{aligned}$$

Just for kicks, here's the graph with the tangent line.