

Calculus I, Section 3.5, #58  
Implicit Differentiation

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Find the derivative of the function. Simplify where possible.<sup>1</sup>

$$y = \cos^{-1}(\sin^{-1}(t))$$

Let's just do the straightforward derivative:

$$\begin{aligned}\frac{dy}{dt} &= -\frac{1}{\sqrt{1 - (\sin^{-1}(t))^2}} \cdot \frac{d}{dt} [\sin^{-1}(t)] \\ &= -\frac{1}{\sqrt{1 - (\sin^{-1}(t))^2}} \frac{1}{\sqrt{1 - t^2}} \\ &= -\frac{1}{\sqrt{1 - t^2} \sqrt{1 - (\sin^{-1}(t))^2}}\end{aligned}$$

Another possibility is to use implicit differentiation.

$$y = \cos^{-1}(\sin^{-1}(t))$$

so

$$\begin{aligned}\cos(y) &= \cos(\cos^{-1}(\sin^{-1}(t))) \\ \cos(y) &= \sin^{-1}(t)\end{aligned}$$

Now we differentiate both sides

$$\begin{aligned}\frac{d}{dt} [\cos(y)] &= \frac{d}{dt} [\sin^{-1}(t)] \\ -\sin(y) \cdot \frac{dy}{dt} &= \frac{1}{\sqrt{1 - t^2}}\end{aligned}$$

so

$$\frac{dy}{dt} = -\frac{1}{\sin(y) \sqrt{1 - t^2}}$$

Which form of the derivative we choose would depend on how we are going to use it.

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<sup>1</sup>Stewart, *Calculus, Early Transcendentals*, p. 216, #58.