Differentiate the function.[1]

$$H(z) = \ln\left(\sqrt{\frac{a^2 - z^2}{a^2 + z^2}}\right)$$

Let's compute the derivative directly.

$$H(z) = \ln\left(\sqrt{\frac{a^2 - z^2}{a^2 + z^2}}\right)$$

so

$$\begin{aligned}
H'(z) &= \frac{1}{\sqrt{\frac{a^2-z^2}{a^2+z^2}}} \cdot \frac{1}{2} \cdot \left(\frac{a^2 - z^2}{a^2 + z^2}\right)^{-1/2} \cdot \frac{\left(a^2 + z^2\right) \cdot -2z - \left(a^2 - z^2\right) \cdot 2z}{\left(a^2 + z^2\right)^2} \\
&= \frac{1}{2 \cdot \sqrt{\frac{a^2-z^2}{a^2+z^2}}} \cdot \frac{1}{\sqrt{\frac{a^2-z^2}{a^2+z^2}}} \cdot \frac{-2a^2z - 2z^3 - 2a^2z + 2z^3}{\left(a^2 + z^2\right)^2} \\
&= \frac{1}{2 \cdot \frac{a^2-z^2}{a^2+z^2}} \cdot \frac{-4a^2z}{\left(a^2 + z^2\right)^2} \\
&= \frac{a^2 + z^2}{2\left(a^2 - z^2\right)} \cdot \frac{-4a^2z}{\left(a^2 + z^2\right)^2} \\
&= \frac{-2a^2z}{\left(a^2 - z^2\right)\left(a^2 + z^2\right)}
\end{aligned}$$

or

$$= \frac{-2a^2z}{a^4 - z^4}$$

or

$$= \frac{2a^2z}{z^4 - a^4}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 223, #20.

Continued $\Longrightarrow$

We could also apply properties of logarithms first, then compute the derivative.

$$\begin{aligned}
H(z) &= \ln\left(\sqrt{\frac{a^2 - z^2}{a^2 + z^2}}\right) \\
&= \frac{1}{2}\ln\left(\frac{a^2 - z^2}{a^2 + z^2}\right) \\
&= \frac{1}{2}\left(\ln\left(a^2 - z^2\right) - \ln\left(a^2 + z^2\right)\right) \\
&= \frac{1}{2}\ln\left(a^2 - z^2\right) - \frac{1}{2}\ln\left(a^2 + z^2\right)
\end{aligned}$$

so

$$\begin{aligned}
H'(z) &= \frac{1}{2} \cdot \frac{1}{a^2 - z^2} \cdot -2z - \frac{1}{2} \cdot \frac{1}{a^2 + z^2} \cdot 2z \\
&= \frac{-z\left(a^2 + z^2\right) - z\left(a^2 - z^2\right)}{\left(a^2 - z^2\right)\left(a^2 + z^2\right)} \\
&= \frac{-2a^2 z}{\left(a^2 - z^2\right)\left(a^2 + z^2\right)}
\end{aligned}$$

or

$$= \frac{-2a^2 z}{a^4 - z^4}$$

or

$$= \frac{2a^2 z}{z^4 - a^4}$$