

Calculus I, Section 3.6, #26
Derivatives of Logarithmic Functions

Find y' and y'' .¹

$$y = \ln(1 + \ln(x))$$

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so

$$\begin{aligned} y' &= \frac{1}{1 + \ln(x)} \cdot \left(0 + \frac{1}{x}\right) \\ &= \frac{1}{x(1 + \ln(x))} \end{aligned}$$

so

$$\begin{aligned} y'' &= -\frac{1}{(x(1 + \ln(x)))^2} \cdot \frac{d}{dx} [x(1 + \ln(x))] && \text{Since } \frac{d}{dx} \left[\frac{1}{\text{stuff}} \right] = -\frac{1}{(\text{stuff})^2} \cdot \frac{d\text{stuff}}{dx} \\ &= -\frac{1}{(x(1 + \ln(x)))^2} \cdot \left(x \cdot \left(0 + \frac{1}{x}\right) + (1 + \ln(x)) \cdot 1 \right) \\ &= -\frac{1}{(x(1 + \ln(x)))^2} \cdot (0 + 1 + 1 + \ln(x)) \\ &= -\frac{2 + \ln(x)}{x^2(1 + \ln(x))^2} \end{aligned}$$

¹Stewart, *Calculus, Early Transcendentals*, p. 223, #26.