Calculus I, Section 3.6, #44
Derivatives of Logarithmic Functions

Use logarithmic differentiation to find the derivative of the function.[1]

$$y = x^{\cos(x)}$$

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so

$$\begin{aligned}\ln(y) &= \ln\left(x^{\cos(x)}\right)\\ \ln(y) &= \cos(x)\cdot\ln(x)\end{aligned}$$

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$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(y)\right] &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\cos(x)\cdot\ln(x)\right]\\ \frac{1}{y}\cdot\frac{\mathrm{d}y}{\mathrm{d}x} &= \cos(x)\cdot\frac{1}{x}+\ln(x)\cdot-\sin(x)\\ y\cdot\frac{1}{y}\cdot\frac{\mathrm{d}y}{\mathrm{d}x} &= y\left(\cos(x)\cdot\frac{1}{x}+\ln(x)\cdot-\sin(x)\right)\\ \frac{\mathrm{d}y}{\mathrm{d}x} &= y\left(\frac{\cos(x)}{x}-\sin(x)\ln(x)\right)\end{aligned}$$

so

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= x^{\cos(x)}\left(\frac{\cos(x)}{x}-\frac{x\sin(x)\ln(x)}{x}\right)\\ \frac{\mathrm{d}y}{\mathrm{d}x} &= x^{\cos(x)}\left(\frac{\cos(x)-x\sin(x)\ln(x)}{x}\right)\end{aligned}$$

or

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{\cos(x)-1}\left(\cos(x)-x\sin(x)\ln(x)\right)$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 223, #44.