A particle moves with position function[1]

$$s = t^4 - 4t^3 - 20t^2 + 20t \qquad t \geq 0$$

(a) *At what times does the particle have a velocity of* $20\,\mathrm{m/s}$*?* Velocity $v(t)$ is the derivative of the position function $s(t)$, so we compute

$$v(t) = s'(t) = 4t^3 - 12t^2 - 40t + 20$$

and we solve

$$\begin{aligned} 20 &= 4t^3 - 12t^2 - 40t + 20 \\ 0 &= 4t^3 - 12t^2 - 40t \\ 0 &= 4t\left(t^2 - 3t - 10\right) \\ 0 &= 4t\,(t+2)\,(t-5) \end{aligned}$$

By zero product property, $t = 0$, $t = -2$, and $t = 5$. But we are to;d $t \geq 0$. Thus, the particle has velocity of $20\,\mathrm{m/s}$ at $t = 0$ and $t = 5$ seconds.

(b) *At what time is the acceleration* 0*? What is the significance of this value of* t*?*

Acceleration $a(t)$ is the derivative of the velocity function $v(t)$, so we compute

$$a(t) = v'(t) = 12t^2 - 24t - 40$$

and we solve

$$\begin{aligned} 0 &= 12t^2 - 24t - 40 \\ 0 &= 4\left(3t^2 - 6t - 10\right) \end{aligned}$$

The quadratic expression on the right does not factor, so we'll use quadratic formula to solve the equation

$$0 = 3t^2 - 6t - 10$$

so

$$\begin{aligned} t &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4\,(3)\,(-10)}}{2\,(3)} \\ &= \frac{6 \pm \sqrt{156}}{6} \end{aligned}$$

so

$$t \approx 3.08 \quad \text{or} \quad t \approx -1.08$$

Here, $t \approx -1.08$ is out of the domain, so the acceleration is zero when $t \approx 3.08\,\mathrm{s}$. Note that before $3.08\,\mathrm{s}$ the acceleration is negative so the particle is slowing down, and after $3.08\,\mathrm{s}$ the acceleration is positive, so the particle is speeding up. Thus, $t = 3.08\,\mathrm{s}$ is the time for which the velocity is a minimum.

[1] Stewart, *Calculus, Early Transcendentals*, p. 233, #10.