(a) *The volume of a growing spherical cell is* $V = \frac{4}{3}\pi r^3$, *where the radius* r *is measured in micrometers* ($1\,\mu\text{m} = 10^{-6}\,\text{m}$). *Find the average rate of change (AROC) of* V *with respect tor when* r *changes from*[1]

(i) 5 *to* $8\,\mu$m

$$\begin{aligned}
\text{AROC}_{5\to8} &= \frac{V(8)-V(5)}{8-5} \\
&= \frac{\frac{4}{3}\pi\,(8)^3 - \frac{4}{3}\pi\,(5)^3}{8-5} \\
&= \frac{\frac{4}{3}\pi\,(512-125)}{3} \\
&= \frac{4}{9}\cdot\pi\cdot 387 \\
&= 172\pi \\
&\approx 540.35\,\mu\text{m}^3/\mu\text{m}
\end{aligned}$$

(ii) 5 *to* $6\,\mu$m

$$\begin{aligned}
\text{AROC}_{5\to6} &= \frac{V(6)-V(5)}{6-5} \\
&= \frac{\frac{4}{3}\pi\,(6)^3 - \frac{4}{3}\pi\,(5)^3}{6-5} \\
&= \frac{\frac{4}{3}\pi\,(216-125)}{1} \\
&= \frac{4}{3}\cdot\pi\cdot 91 \\
&= \frac{364}{3}\pi \\
&\approx 381.18\,\mu\text{m}^3/\mu\text{m}
\end{aligned}$$

(ii) 5 *to* $5.1\,\mu$m

$$\begin{aligned}
\text{AROC}_{5\to5.1} &= \frac{V(5.1)-V(5)}{5.1-5} \\
&= \frac{\frac{4}{3}\pi\,(5.1)^3 - \frac{4}{3}\pi\,(5)^3}{5.1-5} \\
&= \frac{\frac{4}{3}\pi\,(132.651-125)}{0.1} \\
&= \frac{4}{3}\cdot\pi\cdot 76.51 \\
&\approx 320.48\,\mu\text{m}^3/\mu\text{m}
\end{aligned}$$

(b) *Find the instantaneous rate of change of* V *with respect to* r *when* $r = 5\,\mu\text{m}^3/\mu\text{m}$.

$V'(r) = \frac{4}{3}\pi\cdot 3r^2 = 4\pi r^2$, so $V'(5) = 100\pi \approx 314.16\,\mu\text{m}^3/\mu\text{m}$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 234, #16.

Continued $\Longrightarrow$

(c) *Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area. Explain geometrically why this result is true.*

$V'(r) = \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2$, which is the formula for the surface area of a sphere. As we increase the radius by a tiny bit Δr, the increase in the volume will be about the surface area times the thickness of the increase. Thus

$$\Delta V \approx 4\pi r^2 \cdot \Delta r \text{ so } \frac{\Delta V}{\Delta r} \approx 4\pi r^2$$