Some of the highest tides in the world occur in the Bay of Fundy on the Atlantic Coast of Canada. At Hopewell Cape the water depth at low tide is about 2.0 m *and at high tide it is about* 12.0 m. *The natural period of oscillation is a little more than 12 hours and on June 30,2009, high tide occurred at 6:45* AM. *This helps explain the following model for the water depth D (in meters) as a function of time t (in hours after midnight) on that day:*

$$D(t) = 7 + 5\cos\left[0.503\,(t - 6.75)\right]$$

How fast was the tide rising (or falling) at the following times?[1]

(a) 3:00 AM

(b) 6:00 AM

(c) 9:00 AM

(d) Noon

To answer "how fast" requires the derivative.

$$\begin{aligned} D(t) &= 7 + 5\cos\left[0.503\,(t - 6.75)\right] \\ D(t) &= 7 + 5\cos\left[0.503t - 3.39525\right] \end{aligned}$$

so

$$\begin{aligned} \frac{\mathrm{d}D}{\mathrm{d}t} &= 0 + 5 \cdot -\sin\left[0.503t - 3.39525\right] \cdot 0.503t \\ \frac{\mathrm{d}D}{\mathrm{d}t} &= -2.515\sin\left[0.503t - 3.39525\right] \end{aligned}$$

Note that the units on $\frac{\mathrm{d}D}{\mathrm{d}t}$ are $\frac{\mathrm{m}}{\mathrm{hr}}$.

Now we use the TI-84 to get

$$\begin{aligned} \frac{\mathrm{d}D}{\mathrm{d}3} &= -2.515\sin\left[0.503\,(3) - 3.39525\right] \\ &\approx 2.39 \\ \frac{\mathrm{d}D}{\mathrm{d}6} &= -2.515\sin\left[0.503\,(6) - 3.39525\right] \\ &\approx 0.93 \\ \frac{\mathrm{d}D}{\mathrm{d}9} &= -2.515\sin\left[0.503\,(9) - 3.39525\right] \\ &\approx -2.28 \\ \frac{\mathrm{d}D}{\mathrm{d}12} &= -2.515\sin\left[0.503\,(12) - 3.39525\right] \\ &\approx -1.21 \end{aligned}$$

where the positive derivatives indicate the tide is rising and the negative derivatives indicate the tide is falling.

[1] Stewart, *Calculus, Early Transcendentals*, p. 234, #22.