The cost function for a certain commodity is[1]

$$C(q) = 84 + 0.16q - 0.0006q^2 + 0.000003q^3$$

(a) *Find and interpret* $C'(100)$.

Here, $C'(t) = 0.16 - 0.0012q + 0.000009q^2$. This gives the rate of change of the cost of production with respect to the quantity produced. That is, $C'(t)$ estimates the cost to produce the $(t+1)$st item.

$$C'(100) = 0.16 - 0.0012\,(100) + 0.000009\,(100)^2 = 0.13$$

Thus, the cost to produce the 101st item is approximately \$0.13.

(b) *Compare* $C'(100)$ *with the cost of producing the 101st item.*

The actual cost to produce the 101st item is given by $C(101) - C(100)$. Using the TI-84, we get

$$\begin{aligned} C(101) - C(100) &= \left(84 + 0.16\,(101) - 0.0006\,(101)^2 + 0.000003\,(101)^3\right) - \\ &\qquad \left(84 + 0.16\,(100) - 0.0006\,(100)^2 + 0.000003\,(100)^3\right) \\ &= 0.13030299 \end{aligned}$$

Thus the marginal cost is simple to compute and gives a very close estimate to the actual cost of producing the 101st item.

[1] Stewart, *Calculus, Early Transcendentals*, p. 236, #32.