Strontium-90 has a half-life of 28 days.[1]

(a) *A sample has a mass of* 50 mg *initially. Find a formula for the mass remaining after* t *days.*

Let $m(t)$ represent the mass of strontium-90 in milligrams at time t days. Then we have the function

$$m(t) = m_0 e^{kt}$$

where m_0 is the mass at time $t = 0$.

Substituting the given initial value, we get

$$m(t) = 50e^{kt}$$

Since the half-life is 28 days, we know when $t = 28$ there will be only 25 mg of the strontium-90 remaining. Substituting these values into our equation gives

$$\begin{aligned} m(t) &= 50e^{kt} \\ 25 &= 50e^{k28} \\ \frac{1}{2} &= e^{k\cdot 28} \\ \ln\left(\tfrac{1}{2}\right) &= \ln\left(e^{28k}\right) \\ \ln\left(\tfrac{1}{2}\right) &= 28k \cdot \ln(e) \\ \frac{\ln(1/2)}{28} &= k \\ -0.0248 &\approx k \end{aligned}$$

Thus the formula is

$$m(t) = 50e^{-0.0248t}$$

(b) *Find the mass remaining after 40 days.* Using our formula from part (a),

$$\begin{aligned} m(40) &= 50e^{-0.0248\cdot 40} \\ m(40) &\approx 18.54\,\text{mg} \end{aligned}$$

(c) *How long does it take the sample to decay to a mass of* 2 mg*?* We solve

$$\begin{aligned} 2 &= 28e^{-0.0248t} \\ \frac{2}{28} &= e^{-0.0248t} \\ \ln\left(\frac{1}{14}\right) &= \ln\left(e^{-0.0248t}\right) \\ \ln(1/14) &= -0.0248t \cdot \ln(e) \\ \frac{\ln(1/14)}{-0.0248} &= t \\ 106.41 &\approx t \end{aligned}$$

Thus, after about 106 days, the amount of strontium-90 remaining will be 2 mg.

[1] Stewart, *Calculus, Early Transcendentals*, p. 243, #8.

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(d) *Sketch the graph of the mass function.*