In a murder investigation, the temperature of the corpse was 32.5 °C *at 1:30*PM *and* 30.3 °C *an hour later. Normal body temperature is* 37 °C *and the temperature of the surroundings was* 20 °C. *When did the murder take place?*[1]

We'll create a function giving the body's temperature after 1:30PM. We use this approach because we know the temperature at 1:30PM and we know the temperature one hour later.

Let $T(t)$ be the temperature of the body t hours after 1:30PM. Then we know $T(0) = 32.5$ and $T(1) = 30.3$. We also know $T_s = 20$ and we want the time t_m when $T(t_m) = 37$.

We have Newton's Law of Cooling

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k\,(T - T_s)$$

and substituting

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k\,(T - 20)$$

Now we let $y = T - 20$, so $y(0) = T(0) - 20$ or $y(0) = 32.5 - 20 = 12.5$. This gives us y as the solution to $\mathrm{d}y/\mathrm{d}t = ky$ with $y(0) = 12.5$. We know that differential equations of this form have solutions $y(t) = y(0)\,\mathrm{e}^{kt}$. Now we have

$$\begin{aligned} y(t) &= y(0)\,\mathrm{e}^{kt} \\ &= 12.5\mathrm{e}^{kt} \end{aligned}$$

along with

$$y(1) = T(0) - 20 = 30.3 - 20 = 10.3$$

substituting

$$\begin{aligned} 10.3 &= 12.5\mathrm{e}^{k\cdot 1} \\ \frac{10.3}{12.5} &= \mathrm{e}^{k} \\ \ln\left(\tfrac{10.3}{12.5}\right) &= \ln\left(\mathrm{e}^{k}\right) \\ \ln\left(\tfrac{10.3}{12.5}\right) &= kt \\ -0.1936 &\approx k \end{aligned}$$

So we have the function $y(t) = 12.5\mathrm{e}^{-0.1936t}$. We want the time t_m when $T(t_m) = 37$. Using the same reasoning as above, $y(t_m) = T(t_m) - 20$ or $y(t_m) = 37 - 20 = 17$, so we solve

$$\begin{aligned} y(t_m) = 17 &= 12.5\mathrm{e}^{-0.1936t_m} \\ \frac{17}{12.5} &= \mathrm{e}^{-0.1936t_m} \\ \ln(17/12.5) &= -0.1936t_m \\ \frac{\ln(17/12.5)}{-0.1936} &= t_m \\ -1.5882 &\approx t_m \end{aligned}$$

Thus the murder occurred 1.5882 hrs before 1:30PM. 1.5882 hrs ≈ 95 mins, so the murder occurred at about 11:55PM.

Note: We haven't found the function $T(t)$. Doing so would've helped us avoid the back-and-forth between $T(t)$ and $y(t)$. We will learn how to directly solve for $T(t)$ in Calc II or a differential equations course.

[1] Stewart, *Calculus, Early Transcendentals*, p. 244, #16.