

Calculus I, Section 3.8, #20
Exponential Growth and Decay

- (a) If \$1000 is borrowed at 8% interest, find the amounts due at the end of 3 years if the interest is compounded (i) annually, (ii) quarterly, (iii) monthly, (iv) weekly, (v) daily, (vi) hourly, and (vii) continuously.¹

For (i)-(vi), we use the formula

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

where $A_0 = 1000$, $r = 0.08$, $t = 3$ and n is the number of compoundings per year. (We'll assume 52 weeks in a year and 365 days in a year.)

(i) If $n = 1$, $A(3) = 1000 \left(1 + \frac{0.08}{1}\right)^{1 \cdot 3} \approx \1259.71 .

(ii) If $n = 4$, $A(3) = 1000 \left(1 + \frac{0.08}{4}\right)^{4 \cdot 3} \approx \1268.24 .

(iii) If $n = 12$, $A(3) = 1000 \left(1 + \frac{0.08}{12}\right)^{12 \cdot 3} \approx \1268.24 .

(iv) If $n = 52$, $A(3) = 1000 \left(1 + \frac{0.08}{52}\right)^{52 \cdot 3} \approx \1271.01 .

(v) If $n = 365$, $A(3) = 1000 \left(1 + \frac{0.08}{365}\right)^{365 \cdot 3} \approx \1271.22 .

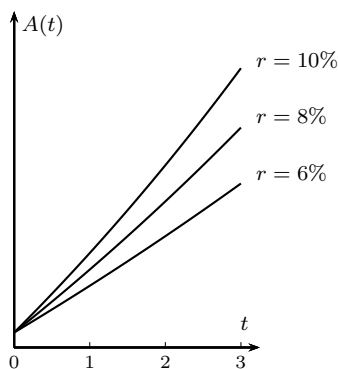
(vi) If $n = 8760$, $A(3) = 1000 \left(1 + \frac{0.08}{8760}\right)^{8760 \cdot 3} \approx \1271.25 .

For (vii), we use the formula $A(t) = A_0 e^{rt}$, so

(vii) $A(3) = 1000e^{0.08 \cdot 3} \approx \1271.25 .

- (b) Suppose \$1000 is borrowed and interest is computed continuously. If $A(t)$ is the amount due after t years, where $0 \leq t \leq 3$, graph $A(t)$ for each of the interest rates 6%, 8%, and 10% on a common screen.

Here, $A_0 = 1000$ and $0 \leq t \leq 3$. We graph $A(t) = 1000e^{r \cdot t}$ for each of the three values of r .



¹Stewart, *Calculus, Early Transcendentals*, p. 244, #20.