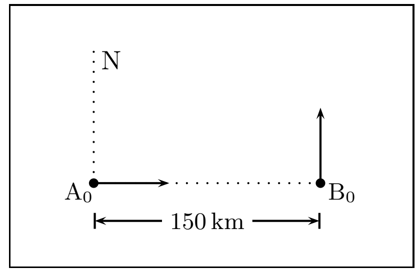
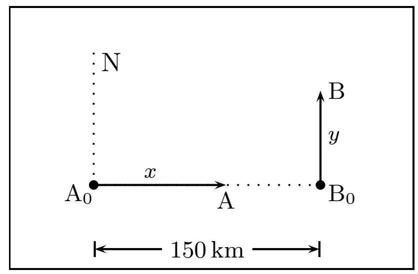
At noon, ship A is 150 km *west of ship B. Ship A is sailing east at* 35 km/h *and ship B is sailing north at* 25 km/h. *How fast is the distance between the ships changing at 4:00*PM*?*[1]

At right is a sketch of the situation at noon. We'll let A_0 be the position of ship A at noon, and B_0 be the position of ship B at noon. (This picture is only valid at noon!)



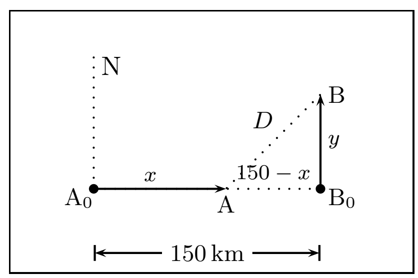
As time moves forward, A moves east at 35 km/h. Let x = distance traveled by A at time t. Similarly, B moves north at 25 km/h. Let y = distance traveled by B at time t.

At right is a new sketch showing the situation at anytime after noon.



We know x is increasing at 35 km/h, so $\frac{dx}{dt} = 35$. Likewise, y is increasing at 25 km/h, so $\frac{dy}{dt} = 25$. Since A has sailed east, the remaining distance to B_0 is $150 - x$.

At right is an updated sketch where we've called the distance between the ships D.



So we want $\frac{dD}{dt}$ at 4:00PM; this corresponds to $t = 4$.

Now $\triangle AB_0B$ is a right triangle, so we'll apply the Pythagorean Theorem

$$D^2 = (150 - x)^2 + y^2$$

or

$$D = \sqrt{(150 - x)^2 + y^2} \qquad \left(\text{Since } D \text{ is a length, we know } D \geq 0 \text{ and } \sqrt{D^2} = D.\right)$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 249, #16.

Continued ⟹

We could compute $\frac{dD}{dt}$ from this, but it will be simpler to use

$$D^2 = (150 - x)^2 + y^2$$

We differentiate with respect to t,

$$\begin{aligned}
\frac{d}{dt}\left[D^2\right] &= \frac{d}{dt}\left[(150 - x)^2\right] + \frac{d}{dt}\left[y^2\right] \\
2D \cdot \frac{dD}{dt} &= 2\,(150 - x)\,(-1) \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} \\
D\frac{dD}{dt} &= -\,(150 - x)\,\frac{dx}{dt} + y\frac{dy}{dt}
\end{aligned}$$

At $t = 4$,

$$\begin{aligned}
x &= 4 \cdot 35 = 140 \\
y &= 4 \cdot 25 = 100
\end{aligned}$$

and we use these values to find D

$$\begin{aligned}
D &= \sqrt{(150 - 140)^2 + (100)^2} \\
&= \sqrt{10^2 + 100^2} \\
&= \sqrt{10{,}100}
\end{aligned}$$

Finally, we substitute and solve for $\frac{dD}{dt}$.

$$\begin{aligned}
D\frac{dD}{dt} &= -\,(150 - x)\,\frac{dx}{dt} + y\frac{dy}{dt} \\
\sqrt{10{,}100}\frac{dD}{dt} &= -\,(150 - 140) \cdot 35 + 100 \cdot 25 \\
\frac{dD}{dt} &= \frac{-350 + 2500}{\sqrt{10{,}100}} \\
&\approx 21.39
\end{aligned}$$

Thus, at 4:00PM, the distance between the ships is increasing at the rate of about 21.39 km/h.