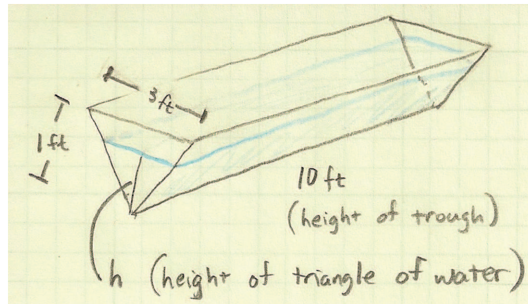
A trough is 10 ft *long and its ends have the shape of isosceles triangles that are* 3 ft *across the top and have a height of* 1 ft. *If the trough is being filled with water at the rate of* $12\,\text{ft}^3/\text{min}$, *how fast is the water level rising when the water is* 6 inches *deep?*[1]



Let $V(t)$ = the volume of water in the trough at time t. We know $\frac{dV}{dt} = 12\,\text{ft}^3/\text{min}$ since the amount of water in the trough is increasing ("filled") at the rate of $12\,\text{ft}^3/\text{min}$.

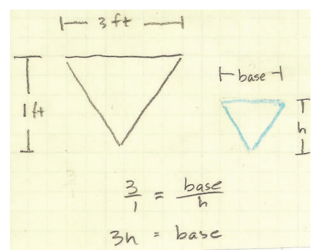
Let h = the "depth" of water in the trough at time t. Note that this depth is in fact the height of the triangle that the water makes on the end of the trough.

We want $\frac{dh}{dt}$ when $h = \frac{1}{2}$ ft.

We need an equation relating V and h.

$$\begin{aligned} V &= (\text{area of base of trough})\,(\text{height of trough}) \\ &= (\text{area of the } \triangle \text{ of water})\,(10\,\text{ft}) \\ &= \left(\frac{1}{2} \cdot \text{base of the } \triangle \text{ of water} \cdot \text{height of the } \triangle \text{ of water}\right)(10\,\text{ft}) \end{aligned}$$

To find the base of the $\triangle$ of water in terms of h, we use the similar triangles shown at right.



$$\begin{aligned} &= \left(\frac{1}{2} \cdot 3h \cdot h\right)(10) \\ &= 15h^2 \end{aligned}$$

We differentiate with respect to t.

$$\begin{aligned} \frac{d}{dt}[V] &= \frac{d}{dt}\left[15h^2\right] \\ \frac{dV}{dt} &= 30h \cdot \frac{dh}{dt} \end{aligned}$$

substituting

$$\begin{aligned} 12 &= 30 \cdot \frac{1}{2}\frac{dh}{dt} \\ \frac{12}{15} &= \frac{dh}{dt} \end{aligned}$$

or

$$\frac{4}{5} = \frac{dh}{dt}$$

Thus the water level is rising at the rate of $\frac{4}{5}$ ft/min when the water is 6 inches deep.

[1] Stewart, *Calculus, Early Transcendentals*, p. 250, #26.