When blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:

$$F = kR^4$$

(This is known as Poiseuille's Law; we will show why it is true in Calc II.) A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow.

Show that the relative change in F is about four times the relative change in R. How will a 5% increase in the radius affect the flow of blood?[1]

The relative change in F is given by $\frac{\mathrm{d}F}{F}$ and the relative change in R is given by $\frac{\mathrm{d}R}{R}$.

$$\begin{aligned} F &= kR^4 \\ \frac{\mathrm{d}F}{\mathrm{d}R} &= 4kR^3 \end{aligned}$$

so

$$\mathrm{d}F = 4kR^3\mathrm{d}R$$

Because we want the relative change in F, we divide by F to get

$$\begin{aligned} \frac{\mathrm{d}F}{F} &= \frac{4kR^3}{F}\mathrm{d}R \\ \frac{\mathrm{d}F}{F} &= \frac{4kR^3}{kR^4}\mathrm{d}R \\ \frac{\mathrm{d}F}{F} &= \frac{4}{R}\mathrm{d}R \\ \frac{\mathrm{d}F}{F} &= 4 \cdot \frac{\mathrm{d}R}{R} \end{aligned}$$

Thus, the relative change of the flux F is four times the relative change in the radius R. A 5% increase in radius corresponds to a 20% increase in blood flow.

[1] Stewart, *Calculus, Early Transcendentals*, p. 257, #40.