**Calculus I, Section 3.11, #23a, #23b**
**Hyperbolic Functions**

*Use the definitions of the hyperbolic functions to find each of the following limits.*[1]

(a) $\lim_{x\to\infty} \tanh(x)$

$$\begin{aligned}
\lim_{x\to\infty} \tanh(x) &= \lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
&= \lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{e^{-x}}{e^{-x}} \\
&= \lim_{x\to\infty} \frac{1 - e^{-2x}}{1 + e^{-2x}} \\
&= \lim_{x\to\infty} \frac{1 - \frac{1}{e^{2x}}}{1 + \frac{1}{e^{2x}}}
\end{aligned}$$

As $x \to \infty$, $\frac{1}{e^{2x}} \to 0$, so we have

$$\begin{aligned}
&= \frac{1-0}{1+0} \\
&= 1
\end{aligned}$$

(b) $\lim_{x\to-\infty} \tanh(x)$

$$\begin{aligned}
\lim_{x\to-\infty} \tanh(x) &= \lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
&= \lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{e^{x}}{e^{x}} \\
&= \lim_{x\to-\infty} \frac{e^{2x} - 1}{e^{2x} + 1}
\end{aligned}$$

As $x \to -\infty$, $e^{2x} \to 0$, so we have

$$\begin{aligned}
&= \frac{0-1}{0+1} \\
&= -1
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 264, #23.