A model for the velocity of a falling object after time t is

$$v(t) = \sqrt{\frac{mg}{k}} \tanh\left(t\sqrt{\frac{gk}{m}}\right)$$

where m is the mass of the object, $g = 9.8\,\mathrm{m/s^2}$ is the acceleration due to gravity, k is a constant, t is measured in seconds, and v in $\mathrm{m/s}$.[1]

(a) Calculate the terminal velocity of the object, that is, $\lim_{t\to\infty} v(t)$.

$$\begin{aligned}\lim_{t\to\infty} v(t) &= \lim_{t\to\infty} \sqrt{\frac{mg}{k}} \tanh\left(t\sqrt{\frac{gk}{m}}\right) \\ &= \sqrt{\frac{mg}{k}} \cdot \lim_{t\to\infty} \tanh\left(t\sqrt{\frac{gk}{m}}\right) \\ &= \sqrt{\frac{mg}{k}} \cdot 1 \qquad \text{(From a previous problem.)} \\ &= \sqrt{\frac{mg}{k}}\end{aligned}$$

(b) If a person falls from a building, the value of the constant k depends on his or her position. For a "belly-to-earth" position, $k = 0.515\,\mathrm{kg/s}$, but for a "feet-first" position, $k = 0.067\,\mathrm{kg/s}$. If a $60\,\mathrm{kg}$ person falls in belly-to-earth position, what is the terminal velocity? What about feet-first?

$$\begin{aligned}\text{terminal velocity}_{belly-to-earth} &= \sqrt{\frac{60 \cdot 9.8}{0.515}} \\ &\approx 33.79\,\mathrm{m/s} \quad \text{(This is about 75.59 miles/h.)}\end{aligned}$$

$$\begin{aligned}\text{terminal velocity}_{feet-first} &= \sqrt{\frac{60 \cdot 9.8}{0.067}} \\ &\approx 93.68\,\mathrm{m/s} \quad \text{(This is about 209.56 miles/h.)}\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 265, #54.