

Find the critical numbers of the function.¹

$$h(p) = \frac{p-1}{p^2+4}$$

The critical numbers are the values of the input variable, p , for which the derivative is zero or undefined. We compute

$$\begin{aligned} h'(p) &= \frac{(p^2+4) \cdot 1 - (p-1) \cdot 2p}{(p^2+4)^2} \\ &= \frac{(p^2+4) - (2p^2-2p)}{(p^2+4)^2} \\ &= \frac{-p^2+2p+4}{(p^2+4)^2} \end{aligned}$$

The numerator doesn't factor, so we solve

$$\begin{aligned} 0 &= \frac{-p^2+2p+4}{(p^2+4)^2} \\ 0 &= -p^2+2p+4 \end{aligned}$$

and from the quadratic formula, we get

$$\begin{aligned} p &= \frac{-(2) \pm \sqrt{(2)^2 - 4 \cdot -1 \cdot 4}}{2 \cdot -1} \\ &= \frac{-2 \pm \sqrt{20}}{-2} \\ &= \frac{-2 \pm 2\sqrt{5}}{-2} \\ &= \frac{-2(1 \mp \sqrt{5})}{-2} \\ &= 1 \mp \sqrt{5} \end{aligned}$$

Thus, the critical numbers are $p = 1 + \sqrt{5}$ and $p = 1 - \sqrt{5}$. Note that $h'(p)$ is defined for all p , so these are the only critical points.

¹Stewart, *Calculus, Early Transcendentals*, p. 283, #36.