

Calculus I, Section 4.1, #42
Maximum and Minimum Values

Find the critical numbers of the function.¹

$$h(t) = 3t - \arcsin(t)$$

Note that because of the $\arcsin(t)$ in the rule for $h(t)$, the domain of $h(t)$ is the interval $[-1,1]$.

The critical numbers are the values of the input variable, t , for which the derivative is zero or undefined. We compute

$$h'(t) = 3 - \frac{1}{\sqrt{1-t^2}}$$

and we solve

$$\begin{aligned} 0 &= 3 - \frac{1}{\sqrt{1-t^2}} \\ \frac{1}{\sqrt{1-t^2}} &= 3 \\ \left(\frac{1}{\sqrt{1-t^2}}\right)^2 &= 3^2 \\ \frac{1}{1-t^2} &= 9 \\ \frac{1}{9} &= 1-t^2 \\ -\frac{8}{9} &= -t^2 \\ \frac{8}{9} &= t^2 \\ \pm\sqrt{\frac{8}{9}} &= t \end{aligned}$$

so

$$t = -\frac{2\sqrt{2}}{3} \quad \text{or} \quad t = \frac{2\sqrt{2}}{3}$$

Both of these are in the domain of $h(t)$, thus, the critical numbers are $t = -\frac{2\sqrt{2}}{3}$ and $t = \frac{2\sqrt{2}}{3}$.

¹Stewart, *Calculus, Early Transcendentals*, p. 284, #42.