Calculus I, Section 4.1, #60
Maximum and Minimum Values

Find the absolute maximum and absolute minimum values of f on the given interval.[1]

$$f(x) = x\mathrm{e}^{x/2}, \quad [-3,1]$$

Because we are working with a closed interval, we need only to find the critical values of $f(x)$ in that interval, and then compute the function values at those critical points and at the endpoints of the interval.

$$f(x) = x\mathrm{e}^{x/2}$$

so

$$\begin{aligned} f'(x) &= x \cdot \mathrm{e}^{x/2} \cdot \frac{1}{2} + \mathrm{e}^{x/2} \cdot 1 \\ &= \mathrm{e}^{x/2}\left(\frac{x}{2} + 1\right) \end{aligned}$$

and we solve

$$0 = \mathrm{e}^{x/2}\left(\frac{x}{2} + 1\right)$$

$\mathrm{e}^{x/2}$ is never zero, so the only solution(s) are given by

$$\begin{aligned} 0 &= \frac{x}{2} + 1 \\ x &= -2 \end{aligned}$$

Thus, $x = -2$ is the only critcal point.

Now we evaluate $f(x) = x\mathrm{e}^{x/2}$ at the endpoints and the critical point.

$$\begin{aligned} f(-3) &= -3\mathrm{e}^{-3/2} \approx -0.6694 \\ f(-2) &= -2\mathrm{e}^{-2/2} \approx -0.7358 \\ f(1) &= 1\mathrm{e}^{1/2} \approx 1.6487 \end{aligned}$$

On the interval $[-3,1]$, the function $f(x) = x\mathrm{e}^{x/2}$ has an absolute minimum of ≈ -0.7358 that occurs at $x = -2$, and an absolute maximum of ≈ 1.6487 that occurs at $x = 1$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 284, #60.