

Calculus I, Section 4.4, #26
Indeterminate Forms and l'Hospital's Rule

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.¹

$$\lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3}$$

As $u \rightarrow \infty$, $\frac{u}{10} \rightarrow \infty$, $e^{u/10} \rightarrow \infty$, and $u^3 \rightarrow \infty$, so the limit has the indeterminate form $\frac{\infty}{\infty}$ and l'Hospital's rule applies.

$$\begin{aligned} \lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3} &\implies \frac{\infty}{\infty} \\ \stackrel{\text{H}}{=} \lim_{u \rightarrow \infty} \frac{e^{u/10} \cdot \frac{1}{10}}{3u^2} &\implies \frac{\infty}{\infty} \\ \stackrel{\text{H}}{=} \lim_{u \rightarrow \infty} \frac{e^{u/10} \cdot \frac{1}{10} \cdot \frac{1}{10}}{6u} &\implies \frac{\infty}{\infty} \\ \stackrel{\text{H}}{=} \lim_{u \rightarrow \infty} \frac{e^{u/10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}{6} & \end{aligned}$$

For this last limit, the numerator becomes infinitely large while the denominator is a constant. Thus,

$$\lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3} = \infty$$

¹Stewart, *Calculus, Early Transcendentals*, p. 311, #26.