Find the limit. Use l'Hospital's Rule where appropriate.If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.[1]

$$\lim_{x\to 0} \csc(x) - \cot(x)$$

As $x \to 0$, $\csc(x) \to \infty$, and $\cot(x) \to \infty$, so the limit has the indeterminate form $\infty - \infty$.

$$\begin{aligned}
&\lim_{x\to 0} \csc(x) - \cot(x) \\
&= \lim_{x\to 0} \frac{1}{\sin(x)} - \frac{cos(x)}{\sin(x)} \\
&= \lim_{x\to 0} \frac{1 - cos(x)}{\sin(x)} \qquad \Longrightarrow \qquad \frac{0}{0} \\
&\overset{\text{H}}{=} \lim_{x\to 0} \frac{0 + \sin(x)}{\cos(x)} \\
&= \frac{0}{1} \\
&= 0
\end{aligned}$$

Thus, $\lim_{x\to 0} \csc(x) - \cot(x) = 0$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 312, #52.