Find the limit. Use l'Hospital's Rule where appropriate.If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.[1]

$$\lim_{x\to\infty} x^{\mathrm{e}^{-x}}$$

As $x \to \infty$, $\mathrm{e}^{-x} \to 0$, so the limit has the indeterminate form ∞^0. Since l'Hospital's Rule doesn't directly apply, we'll name the value of the derivative, then do algebra to get the limit into the proper form.

Let

$$y = \lim_{x\to\infty} x^{\mathrm{e}^{-x}}$$

then

$$\begin{aligned}
\ln(y) &= \ln\left(\lim_{x\to\infty} x^{\mathrm{e}^{-x}}\right)\\
\ln(y) &= \lim_{x\to\infty} \ln\left(x^{\mathrm{e}^{-x}}\right)\\
\ln(y) &= \lim_{x\to\infty} \mathrm{e}^{-x}\cdot\ln(x) \qquad \Longrightarrow \qquad 0\cdot\infty\\
\ln(y) &= \lim_{x\to\infty} \frac{1}{\mathrm{e}^x}\cdot\ln(x)\\
\ln(y) &= \lim_{x\to\infty} \frac{\ln(x)}{\mathrm{e}^x} \qquad \Longrightarrow \frac{\infty}{\infty}\\
\ln(y) &\overset{\mathrm{H}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\mathrm{e}^x}\\
\ln(y) &= \lim_{x\to\infty} \frac{1}{x\mathrm{e}^x}
\end{aligned}$$

so

$$\ln(y) = 0$$

so

$$\begin{aligned}
y &= \mathrm{e}^0\\
y &= 1
\end{aligned}$$

Thus, $\lim_{x\to\infty} x^{\mathrm{e}^{-x}} = 1$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 312, #64.