A company operates 16 oil wells in a designated area. Each pump, on average, extracts 240 barrels of oil daily. The company can add more wells, but every added well reduces the daily average out of each of the wells by 8 barrels. How many wells should the company add in order to maximize daily production?[1]

We need a function $P(w)$ that gives the daily oil production, P as a function of the number of wells w.

Let $w =$ the number of wells.

Each well will produce $240 - 8\,(w - 16)$ barrels of oil per day. Here, the $w - 16$ gives the number of wells over 16 and $8\,(w - 16)$ gives the reduction in daily barrels produced, where $w \geq 16$. Since there are w wells, we get the production function

$$\begin{aligned} P(w) &= w\,(240 - 8\,(w - 16)) \\ &= w\,(368 - 8w) \end{aligned}$$

so

$$P(w) = 368w - 8w^2$$

We compute $P'(w)$ and solve $P'(w) = 0$ to find any critical numbers.

$$P(w) = 368w - 8w^2$$

so

$$P'(w) = 368 - 16w$$

and we solve

$$\begin{aligned} 368 - 16w &= 0 \\ w &= 23 \end{aligned}$$

Note that $P''(w) = -16 < 0$, so by SDT the critical number 23 gives a local maximum.

Thus, daily production is maximized when the company adds $23 - 16 = 7$ wells.

[1] Stewart, *Calculus, Early Transcendentals*, p. 340, #64.