Find f.[1]

$$f''(t) = t^2 + \frac{1}{t^2}, \quad t > 0, \quad f(2) = 3, \quad f'(1) = 2$$

$$\begin{aligned} f'(t) &= \int f''(t)\ \mathrm{d}t \\ &= \int t^2 + \frac{1}{t^2}\ \mathrm{d}t \\ &= \int t^2 + t^{-2}\ \mathrm{d}t \\ &= \frac{t^3}{3} + \frac{t^{-1}}{-1} + C_1 \\ &= \frac{t^3}{3} - \frac{1}{t^3} + C_1 \end{aligned}$$

We are given $f'(1) = 2$, so we substitute $t = 1$ and $f' = 2$

$$\begin{aligned} 2 &= \frac{(1)^3}{3} - \frac{1}{\cdot (1)^3} + C_1 \\ 2 &= \frac{1}{3} - 1 + C_1 \end{aligned}$$

so

$$\frac{8}{3} = C_1$$

Thus, $f'(t) = \frac{t^3}{3} - \frac{1}{3t^3} + \frac{8}{3}$.

Now we'll repeat this process to find f.

$$\begin{aligned} f(t) &= \int f'(t)\ \mathrm{d}t \\ &= \int \frac{t^3}{3} - \frac{1}{t} + \frac{8}{3}\ \mathrm{d}t \\ &= \int \frac{1}{3}t^3 - \frac{1}{t} + \frac{8}{3}\ \mathrm{d}t \\ &= \frac{1}{3} \cdot \frac{t^4}{4} - \ln(t) + \frac{8}{3}t + C_2 \\ &= \frac{t^4}{12} - \ln(t) + \frac{8}{3}t + C_2 \end{aligned}$$

We are given $f(2) = 3$, so we substitute $t = 2$ and $f = 3$

$$\begin{aligned} 3 &= \frac{(2)^4}{12} - \ln(2) + \frac{8}{3} \cdot 2 + C_2 \\ 3 &= \frac{20}{3} - \ln(2) + C_2 \end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 356, #42.

Continued $\Longrightarrow$

so

$$-\frac{11}{3} + \ln(2) = C_2$$

Thus, $f(t) = \dfrac{t^4}{12} - \ln(t) + \dfrac{8}{3}t - \dfrac{11}{3} + \ln(2)$.