Find f.[1]

$$f''(t) = \sqrt[3]{t} - \cos(t), \quad f(0) = 2, \quad f(1) = 2$$

$$\begin{aligned} f'(t) &= \int f''(t)\ \mathrm{d}t \\ &= \int \sqrt[3]{t} - \cos(t)\ \mathrm{d}t \\ &= \int t^{1/3} - \cos(t)\ \mathrm{d}t \\ &= \frac{t^{4/3}}{4/3} - \sin(t) + C_1 \\ &= \frac{3t^{4/3}}{4} - \sin(t) + C_1 \end{aligned}$$

Note that we cannot substitute at this point becaue neither of our initial conditions include f'.

Now we'll repeat this process to find f.

$$\begin{aligned} f(t) &= \int f'(t)\ \mathrm{d}t \\ &= \int \frac{3t^{4/3}}{4} - \sin(t) + C_1\ \mathrm{d}t \\ &= \frac{3}{4} \cdot \frac{t^{7/3}}{7/3} + \cos(t) + C_1 t + C_2 \\ &= \frac{3}{4} \cdot \frac{3}{7} \cdot t^{7/3} + \cos(t) + C_1 t + C_2 \\ &= \frac{9}{28} t^{7/3} + \cos(t) + C_1 t + C_2 \end{aligned}$$

We are given $f(0) = 2$, so we substitute $t = 0$ and $f = 2$

$$\begin{aligned} 2 &= \frac{9}{28} (0)^{7/3} + \cos(0) + C_1 \cdot 0 + C_2 \\ 2 &= 0 + 1 + 0 + C_2 \end{aligned}$$

so

$$1 = C_2$$

We are given $f(1) = 2$, so we substitute $t = 1$ and $f = 2$

$$\begin{aligned} 2 &= \frac{9}{28} (1)^{7/3} + \cos(1) + C_1 \cdot 1 + 1 \\ 2 &= \frac{9}{28} + \cos(1) + C_1 + 1 \end{aligned}$$

so

$$\begin{aligned} 1 - \frac{9}{28} - \cos(1) &= C_1 \\ \frac{19}{28} - \cos(1) &= C_1 \end{aligned}$$

Thus,

$$f(t) = \frac{9}{28} t^{7/3} + \cos(t) + \left(\frac{19}{28} - \cos(1) \right) t + 1$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 356, #46.