A model rocket is fired vertically upward from rest. Its acceleration for the first three seconds is $a(t) = 60t$, at which time the fuel is exhausted and it becomes a freely "falling" body. Fourteen seconds later, the rocket's parachute opens, and the (downward) velocity slows linearly to -18 ft/s in 5 seconds. The rocket then "floats" to the ground at that rate.[1]

(a) *Determine the position function s and the velocity function v (for all times t.) Sketch the graphs of s and v.*

For $0 \le t \le 3$, we have $a(t) = 60t$, so $v(t) = 30t^2 + C$. Since $v(0) = 0$, we get $C = 0$ and $v(t) = 30t^2$. Then $s(t) = 10t^3 + C$. Since $s(0) = 0$, we get $C = 0$ and $s(t) = 10t^3$.

At $t = 3$, we have $v(3) = 30 \cdot 3^2 = 270$ and $s(3) = 10 \cdot 3^3 = 270$. (This is purely coincidental.)

For $3 \le t \le 17$, we have $a(t) = -g = -32$, so $v(t) = -32\,(t-3) + C$ since this velocity function applies after $t = 3$. Because $v(3) = 270$, we get $270 = -32\,(3-3) + C$ or $C = 270$. Thus

$$v(t) = -32\,(t-3) + 270$$

Again, for $3 \le t \le 17$, we have $s(t) = -16\,(t-3)^2 + 270\,(t-3) + C$ and if we substitute the values for $t = 3$, we get $270 = -16\,(3-3)^2 + 270\,(3-3) + C$ or $C = 270$. Thus,

$$s(t) = -16\,(t-3)^2 + 270\,(t-3) + 270$$

Note that $v(17) = -178$ and $s(17) = 914$.

For $17 \le t \le 22$, the velocity increases linearly from -178 ft/s to -18 ft/sec, so

$$\frac{\Delta v}{\Delta t} = \frac{-18 - (-178)}{22 - 14} = 32$$

Thus $v(t) = 32\,(t-17) - 178$. Computing the antiderivative, we get $s(t) = 16\,(t-17)^2 - 178\,(t-17) + C$. Substituting the values for $t = 17$, we get

$$s(t) = 16\,(t-17)^2 - 178\,(t-17) + 914$$

Note that $s(22) = 424$.

For $t > 22$, we have $v(t) = -18$, so $s(t) = -18\,(t-22) + C$. Substituting the values for $t = 22$, we get

$$s(t) = -18\,(t-22) + 424$$

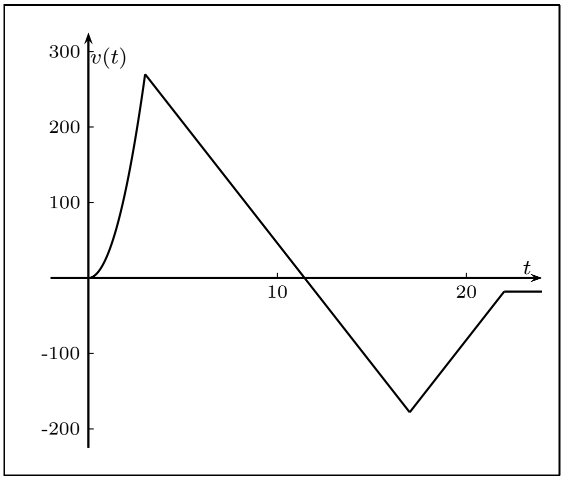
Thus, until the rocket lands,

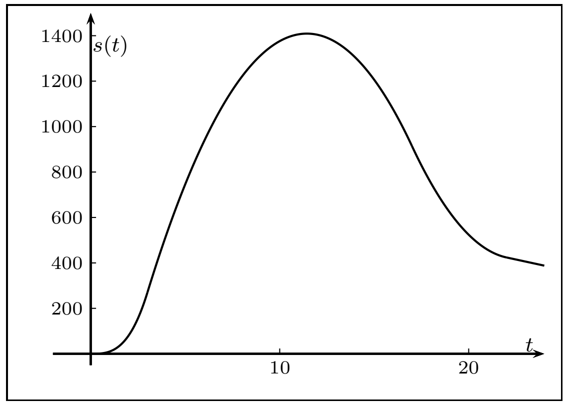
$$v(t) = \begin{cases} 30t^2 & \text{if} \quad 0 \le t \le 3 \\ -32\,(t-3) + 270 & \text{if} \quad 3 \le t \le 17 \\ 32\,(t-17) - 178 & \text{if} \quad 17 \le t \le 22 \\ -18 & \text{if} \quad t > 22 \end{cases}$$

$$s(t) = \begin{cases} 10t^3 & \text{if} \quad 0 \le t \le 3 \\ -16\,(t-3)^2 + 270\,(t-3) + 270 & \text{if} \quad 3 \le t \le 17 \\ 16\,(t-17)^2 - 178\,(t-17) + 914 & \text{if} \quad 17 \le t \le 22 \\ -18\,(t-22) + 424 & \text{if} \quad t > 22 \end{cases}$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 357, #78.

Continued $\Longrightarrow$





(b) *At what time does the rocket reach its maximum height, and what is that height?*

The maximum height will be reached when velocity is zero on $3 \leq t \leq 17$, so we solve

$$\begin{aligned} -32\,(t-3)+270 &= 0 \\ -32t+96+270 &= 0 \\ -32t &= 363 \\ t &= 11.34375 \end{aligned}$$

and the maximum height is given by

$$\begin{aligned} s(11.34375) &= -16\,(11.34375-3)^2+270\,(11.34375-3)+270 \\ &\approx 1409.06\,\text{ft} \end{aligned}$$

(c) *At what time does the rocket land?*

We want the time after launch when the height (position) is zero. We solve

$$\begin{aligned} 0 = s(t) &= -18\,(t-22)+424 \\ 0 &= -18t+396+424 \\ -820 &= -18t \\ t &\approx 45.6\,\text{s} \end{aligned}$$