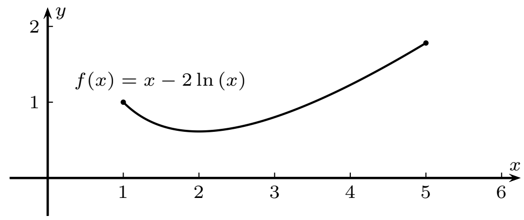
(a) *Graph the function*[1]

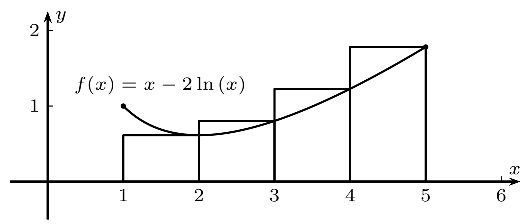
$$f(x) = x - 2\ln(x) \qquad 1 \le x \le 5$$

The function is graphed at right.



(b) *Estimate the area under the graph of f using four approximating rectangles and taking the sample points to be (i) right endpoints and (ii) midpoints. In each case sketch the curve and the rectangles.*

(i) The function is graphed at right with the four right endpoint rectangles.

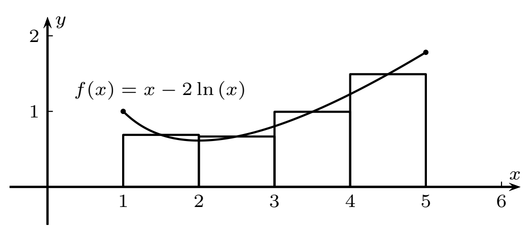


The width of each subinterval is $\Delta x = \frac{5-1}{4} = 1$ so

$$\begin{aligned} \text{Area} &\approx f(2)\cdot\Delta x + f(3)\cdot\Delta x \\ &\quad + f(4)\cdot\Delta x + f(5)\cdot\Delta x \\ &= (f(2)+f(3)+f(4)+f(5))\cdot\Delta x \\ &\approx (0.6137+0.8028+1.2274+1.7811)\cdot 1 \\ &= 4.4250 \end{aligned}$$

Thus, using four right endpoint rectangles, the area under the graph of $f(x) = x - 2\ln(x)$ for $1 \le x \le 5$ is about 4.4250 square units.

(ii) The function is graphed at right with the four rectangles formed using the midpoint of each subinterval.



The width of each subinterval is $\Delta x = \frac{5-1}{4} = 1$ so

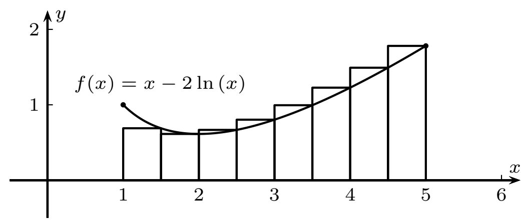
$$\begin{aligned} \text{Area} &\approx f(1.5)\cdot\Delta x + f(2.5)\cdot\Delta x \\ &\quad + f(3.5)\cdot\Delta x + f(4.5)\cdot\Delta x \\ &= (f(1.5)+f(2.5)+f(3.5)+f(4.5))\cdot\Delta x \\ &\approx (0.6891+0.6674+0.9945+1.4918)\cdot 1 \\ &= 3.8428 \end{aligned}$$

Thus, using four midpoint rectangles, the area under the graph of $f(x) = x - 2\ln(x)$ for $1 \le x \le 5$ is about 3.8428 square units.

[1] Stewart, *Calculus, Early Transcendentals*, p. 375, #6.

(c) *Improve your estimates in part (b) by using eight rectangles.*

(i) The function is graphed at right with the eight right endpoint rectangles.

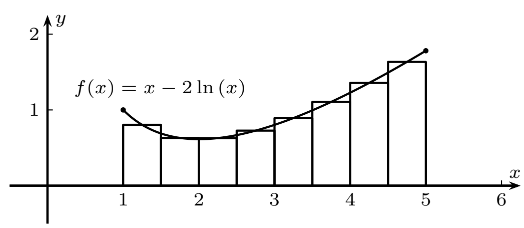


The width of each subinterval is $\Delta x = \frac{5-1}{8} = \frac{1}{2}$ so

$$\begin{aligned}\text{Area} &\approx f(1.5)\cdot\Delta x + f(2)\cdot\Delta x \\ &\quad + f(2.5)\cdot\Delta x + f(3)\cdot\Delta x \\ &\quad + f(3.5)\cdot\Delta x + f(4)\cdot\Delta x \\ &\quad + f(4.5)\cdot\Delta x + f(5)\cdot\Delta x \\ &= (f(1.5)+f(2)+f(2.5)+f(3)+f(3.5)+f(4)+f(4.5)+f(4))\cdot\Delta x \\ &\approx (0.6891+0.6137+0.6674+0.8028+0.9945+1.2274+1.4918+1.7811)\cdot\frac{1}{2} \\ &= 4.1339\end{aligned}$$

Thus, using eight right endpoint rectangles, the area under the graph of $f(x) = x - 2\ln(x)$ for $1 \le x \le 5$ is about 4.1339 square units.

(ii) The function is graphed at right with the eight rectangles formed using the midpoint of each subinterval.



The width of each subinterval is $\Delta x = \frac{5-1}{8} = \frac{1}{2}$ so

$$\begin{aligned}\text{Area} &\approx f(1.25)\cdot\Delta x + f(1.75)\cdot\Delta x \\ &\quad + f(2.25)\cdot\Delta x + f(2.75)\cdot\Delta x \\ &\quad + f(3.25)\cdot\Delta x + f(3.75)\cdot\Delta x \\ &\quad + f(4.25)\cdot\Delta x + f(4.75)\cdot\Delta x \\ &= (f(1.25)+f(1.75)+f(2.25)+f(2.75)+f(3.25)+f(3.75)+f(4.25)+f(4.75))\cdot\Delta x \\ &\approx (0.8037+0.6307+0.6281+0.7268+0.8927+1.1065+1.3561+1.6337)\cdot\frac{1}{2} \\ &= 3.8892\end{aligned}$$

Thus, using eight midpoint rectangles, the area under the graph of $f(x) = x - 2\ln(x)$ for $1 \le x \le 5$ is about 3.8892 square units.