Calculus I, Section 5.1, #31
Areas and Distances

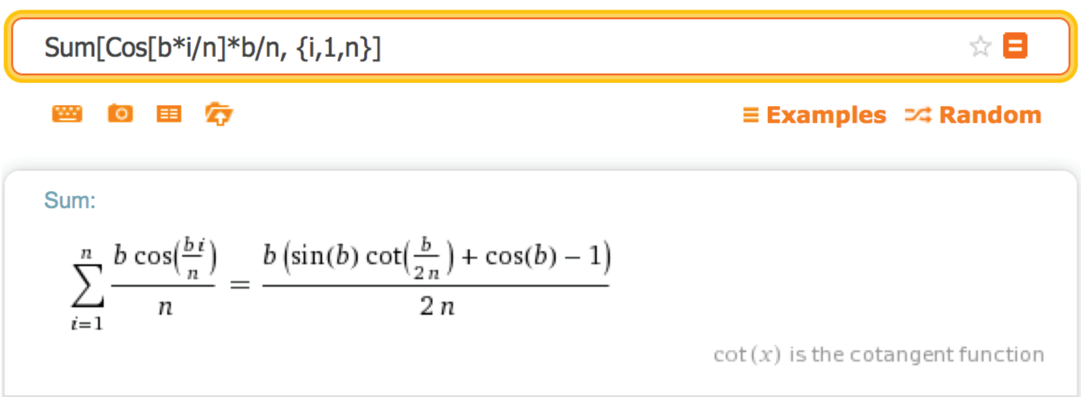
Find the exact area under the cosine curve $y = \cos(x)$ from $x = a$ to $x = b$, where $0 \leq b \leq \frac{\pi}{2}$. (Use a computer algebra system both to evaluate the sum and compute the limit.) In particular, what is the area if[1] *$b = \frac{\pi}{2}$?*

Here we have $y = f(x) = \cos(x)$. Also, $\Delta x = \frac{b-0}{n} = \frac{b}{n}$. We'll use right endpoints (they're usually simpler) so $x_i = 0 + i\Delta x = 0 + i \cdot \frac{b}{n} = \frac{bi}{n}$.

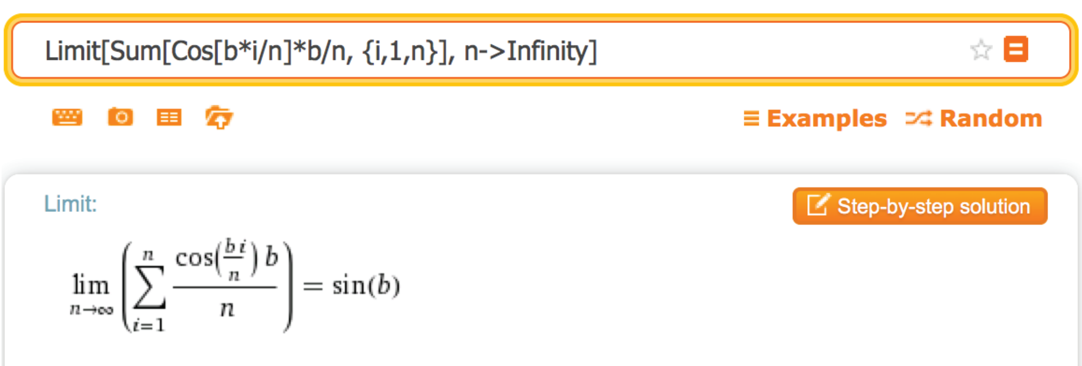
Our equation for area is

$$\begin{aligned} A &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\cos(x_i^*)\,\Delta x\right) \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\cos\left(\frac{bi}{n}\right) \cdot \frac{b}{n}\right) \end{aligned}$$

Using WolframAlpha to evaluate the sum, we get



and then the limit



Thus

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\cos\left(\frac{bi}{n}\right) \cdot \frac{b}{n}\right) = \sin(b)$$

and in particular, if $b = \frac{\pi}{2}$ the area under the cosine curve from $x = 0$ to $x = \frac{\pi}{2}$ is $\sin\left(\frac{\pi}{2}\right) = 1$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 378, #31.