Express the limit as a definite integral on the given interval.[1]

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x, \qquad [1,3]$$

From the definition of the definite integral of f from a to b we have

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \int_a^b f(x)\ \mathrm{d}x$$

and comparing with the given sum

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x = \int_a^b f(x)\ \mathrm{d}x$$

we can see that $f(x) = \dfrac{x}{x^2+4}$, so

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x = \int_a^b \frac{x}{x^2+4}\ \mathrm{d}x$$

Finally, we are given the interval [1,3], thus we have

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x = \int_1^3 \frac{x}{x^2+4}\ \mathrm{d}x$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 389, #20.