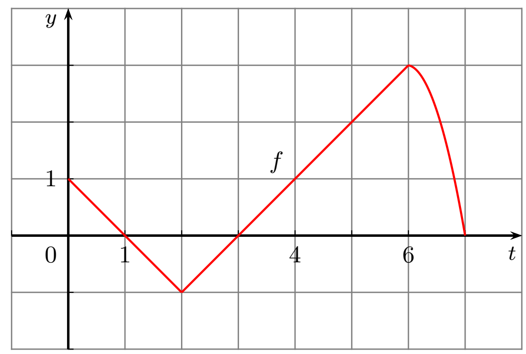
Let $g(x) = \int_0^x f(t)\ \mathrm{d}t$ where f is the function whose graph is shown at right.[1]



(a) *Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5,$ and 6.*

$g(0) = \int_0^0 f(t)\ \mathrm{d}t = 0$ from the property $\int_0^0 f(x)\ \mathrm{d}x = 0$.

$g(1) = \int_0^1 f(t)\ \mathrm{d}t$. From the graph of f, we see that f is nonnegative on the interval [0,1], so the integral is equal to the area between the graph and the x-axis. This region is a triangle, so the area $= \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$. Thus, $g(1) = \frac{1}{2}$.

$g(2) = \int_0^2 f(t)\ \mathrm{d}t$. From the graph of f, we see that f is nonnegative on the interval [0,1], and negative on [1,2]. We know that from 0 to 1, the integral is $\frac{1}{2}$. On the interval [1,2] the area is again $\frac{1}{2}$, but the function is negative, so the *integral* is $-\frac{1}{2}$. Thus $g(2) = \frac{1}{2} - \frac{1}{2} = 0$.

$g(3) = \int_0^3 f(t)\ \mathrm{d}t$. Continuing the same reasoning–integral is positive if function is above the x-axis, integral is negative if function is below the y-axis–we can see $g(3) = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$.

$g(4) = \int_0^4 f(t)\ \mathrm{d}t = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = 0$.

$g(5) = \int_0^5 f(t)\ \mathrm{d}t = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} + \frac{3}{2} = \frac{3}{2}$.

$g(6) = \int_0^6 f(t)\ \mathrm{d}t = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} + \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$.

(b) *Estimate $g(7)$.*

$$\begin{aligned} g(7) &= \int_0^7 f(t)\ \mathrm{d}t \\ &= \int_0^6 f(t)\ \mathrm{d}t + \int_6^7 f(t)\ \mathrm{d}t \end{aligned}$$

and since the area under the graph from 6 to 7 is ≈ 2.2,

$$\begin{aligned} &= 4 + 2.2 \\ &= 6.2 \end{aligned}$$

(c) *Where does g have a maximum value? Where does it have a minimum value?*

g has a maximum value of 6.2 that occurs at $x = 7$. g has a minimum value of $-\frac{1}{2}$ that occurs at $x = 3$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 399, #2.

(d) *Sketch a rough graph of g.*