Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.[1]

$$y = \int_{\sin(x)}^{1} \sqrt{1+t^2}\ \mathrm{d}t$$

Let's remind ourselves of the Fundamental Theorem of Calculus, Part 1:

> **The Fundamental Theorem of Calculus, Part 1** If f is continuous on $[a,b]$, then the function g defined by
>
> $$g(x) = \int_a^x f(t)\ \mathrm{d}t \qquad a \le x \le b$$
>
> is continuous on $[a,b]$ and differentiable on (a,b) and $g'(x) = f(x)$.

First, we'll use properties of the definite integral to make the integral match the form in the Fundamental Theorem.

$$\int_{\sin(x)}^{1} \sqrt{1+t^2}\ \mathrm{d}t = -1 \cdot \int_1^{\sin(x)} \sqrt{1+t^2}\ \mathrm{d}t$$

so we have

$$y = -\int_1^{\sin(x)} \sqrt{1+t^2}\ \mathrm{d}t$$

The minus sign is just a constant factor, so

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -1 \cdot \frac{\mathrm{d}}{\mathrm{d}x}\left[\int_1^{\sin(x)} \sqrt{1+t^2}\ \mathrm{d}t\right]$$

The integral on the right is a function of another function, so we must use chain rule to compute the derivative

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= -1 \cdot \sqrt{1+(\sin(x))^2} \cdot \frac{\mathrm{d}}{\mathrm{d}x}[\sin(x)] \\ &= -1 \cdot \sqrt{1+(\sin(x))^2} \cdot \cos(x)\end{aligned}$$

Thus,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\cos(x)\sqrt{1+\sin^2(x)}$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 400, #18.