*Evaluate the integral.*[1]

$$\int_0^4 (4-t)\sqrt{t}\ \mathrm{d}t$$

We'd like to evaluate this integral using the Fundamental Theorem of Calculus, Part 2 (the option is to compute the limit of the sum).

Let's remind ourselves of the Fundamental Theorem of Calculus, Part 2:

> **The Fundamental Theorem of Calculus, Part 2** If $f$ is continuous on $[a,b]$, then
>
> $$\int_a^b f(x)\ \mathrm{d}x = F(b) - F(a)$$
>
> where $F$ is any antiderivative of $f$, that is, a function such that $F' = f$.

First, we need an antiderivative for the function $(4-t)\sqrt{t}$. This doesn't fit any of the basic patterns we know, but some algebra might help:

$$\begin{aligned}(4-t)\sqrt{t} &= (4-t)\cdot t^{1/2}\\ &= 4t^{1/2} - t\cdot t^{1/2}\\ &= 4t^{1/2} - t^{3/2}\end{aligned}$$

The integrand now fits the basic patterns we know for antiderivatives.

$$\begin{aligned}\int_0^4 (4-t)\sqrt{t}\ \mathrm{d}t &= \int_{t=0}^{t=4} 4t^{1/2} - t^{3/2}\ \mathrm{d}t\\ &= \int_{t=0}^{t=4} 4t^{1/2}\ \mathrm{d}t - \int_{t=0}^{t=4} t^{3/2}\ \mathrm{d}t\\ &= 4\cdot\int_{t=0}^{t=4} t^{1/2}\ \mathrm{d}t - \int_{t=0}^{t=4} t^{3/2}\ \mathrm{d}t\\ &= 4\cdot\left[\frac{t^{1/2+1}}{\frac{1}{2}+1}\right]_{t=0}^{t=4} - \left[\frac{t^{3/2+1}}{\frac{3}{2}+1}\right]_{t=0}^{t=4}\\ &= 4\cdot\left[\frac{t^{3/2}}{\frac{3}{2}}\right]_{t=0}^{t=4} - \left[\frac{t^{5/2}}{\frac{5}{2}}\right]_{t=0}^{t=4}\\ &= 4\cdot\left[\frac{2}{3}\cdot t^{3/2}\right]_{t=0}^{t=4} - \left[\frac{2}{5}\cdot t^{5/2}\right]_{t=0}^{t=4}\\ &= 4\cdot\left[\frac{2}{3}\cdot 4^{3/2} - \frac{2}{3}\cdot 0^{3/2}\right] - \left[\frac{2}{5}\cdot 4^{5/2} - \frac{2}{5}\cdot 0^{5/2}\right]\end{aligned}$$

and since $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$ and $4^{5/2} = \left(\sqrt{4}\right)^5 = 2^5 = 32$,

$$\begin{aligned}&= 4\cdot\left[\frac{2}{3}\cdot 8 - 0\right] - \left[\frac{2}{5}\cdot 32 - 0\right]\\ &= 4\cdot\frac{16}{3} - \frac{64}{5}\\ &= \frac{128}{15}\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 400, #28.