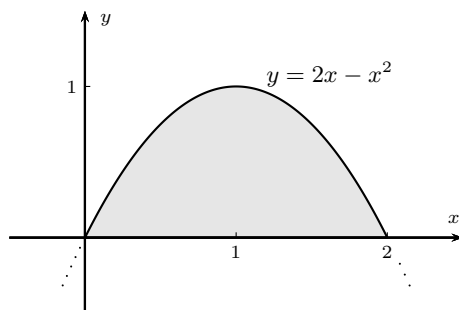


Calculus I, Section 5.3, #48
The Fundamental Theorem of Calculus

Sketch the region enclosed by the given curves and calculate its area.¹

$$y = 2x - x^2, \quad y = 0$$

Here's the graph of the region:



The function $y = 2x - x^2$ is nonnegative on the interval $[0,2]$, *ie.*, the graph is above the x -axis for the entire region, so the area of the region is equal to the definite integral of the function.

$$\begin{aligned} \text{Area} &= \int_0^2 2x - x^2 \, dx \\ &= \left[x^2 - \frac{x^3}{3} \right]_{x=0}^{x=2} \\ &= \left(2^2 - \frac{2^3}{3} \right) - \left(0^2 - \frac{0^3}{3} \right) \\ &= \left(4 - \frac{8}{3} \right) - (0 - 0) \\ &= \left(\frac{12}{3} - \frac{8}{3} \right) - 0 \\ &= \frac{4}{3} \end{aligned}$$

Thus the area of the shaded region is exactly $\frac{4}{3}$.

¹Stewart, *Calculus, Early Transcendentals*, p. 400, #48.