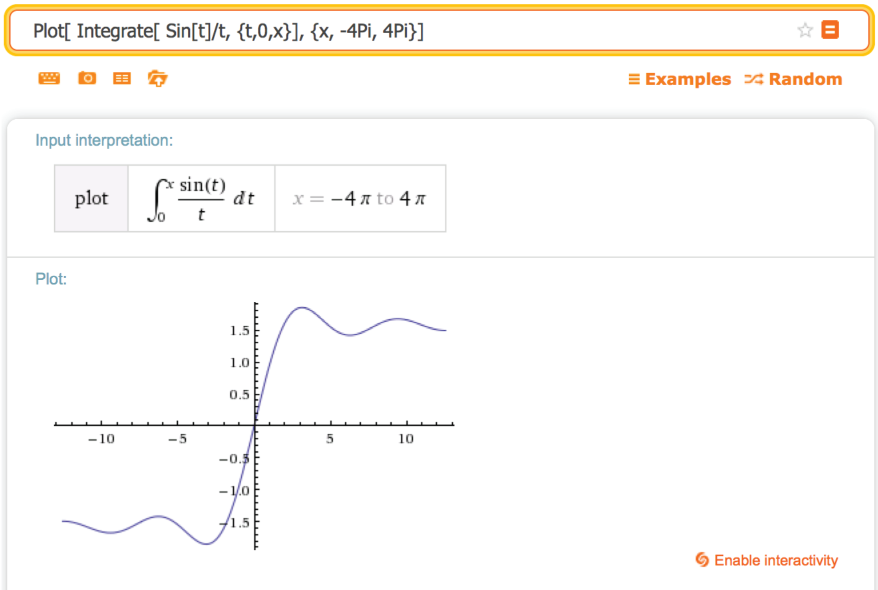
The ***sine integral function***

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t}\,\mathrm{d}t$$

is important in electrical engineering. [The integrand $f(t) = (\sin(t))/t$ is not defined when $t = 0$, but we know its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.][1]

(a) *Draw the graph of* Si. Using WolframAlpha,



(b) *At what values of x does this function have local maximum values?* The function $\mathrm{Si}(x)$ has local maximums at those values of x for which $\mathrm{Si}'(x)$ changes from positive to negative.

We compute the derivative of $\mathrm{Si}(x)$

$$\begin{aligned}\mathrm{Si}'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^x \frac{\sin(t)}{t}\,\mathrm{d}t\right]\\ &= \frac{\sin(x)}{x}\end{aligned}$$

and solve the equation

$$0 = \frac{\sin(x)}{x}$$

and if $x \neq 0$, then

$$0 = \sin(x)$$

So the critical numbers for $\mathrm{Si}(x)$ are $x = n\pi$, where n is an integer.

If $x > 0$, the denominator of $\mathrm{Si}'(x) = \frac{\sin(x)}{x}$ is positive, so the derivative changes from positive to negative wherever $\sin(x)$ changes from positive to negative. From our knowledge of the unit circle, this happens at $x = \pi, 3\pi, 5\pi, \ldots$.

continued...

[1] Stewart, *Calculus, Early Transcendentals*, p. 401, #72.

If $x < 0$, the denominator of $\text{Si}'(x) = \frac{\sin(x)}{x}$ is negative, so the derivative changes from positive to negative wherever $\sin(x)$ changes from negative to positive. From our knowledge of the unit circle, this happens at $x = -2\pi,\ -4\pi,\ -6\pi, \ldots$.

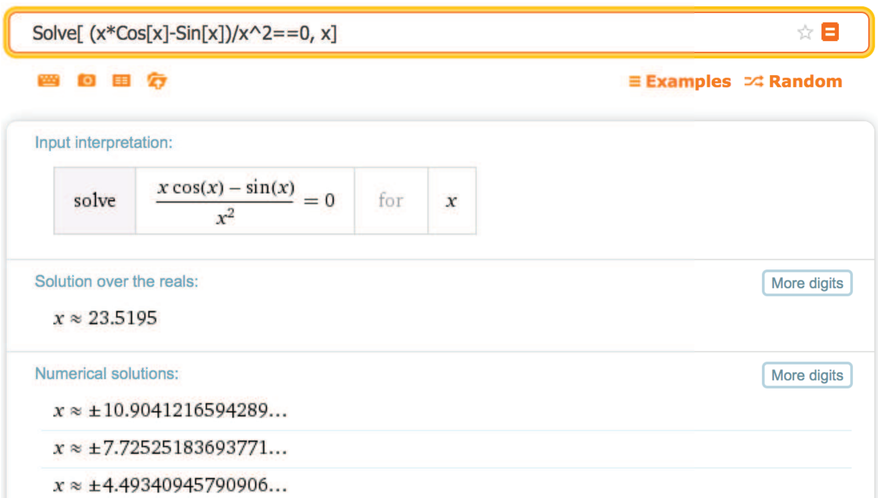
Thus, the local maximums for $\text{Si}(x)$ occur at $x = \ldots, -6\pi,\ -4\pi,\ -2\pi,\ \pi,\ 3\pi,\ 5\pi, \ldots$.

(c) *Find the coordinates of the first inflection point to the right of the origin.*

$$\begin{aligned}\text{Si}''(x) &= \frac{x \cdot \cos(x) - \sin(x) \cdot 1}{x^2} \\ &= \frac{x\cos(x) - \sin(x)}{x^2}\end{aligned}$$

so to find the inflection points, we need to solve the equation $\frac{x\cos(x)-\sin(x)}{x^2} = 0$, and determine if the sign of $func\text{Si}''x$ changes.

Using WolframAlpha,



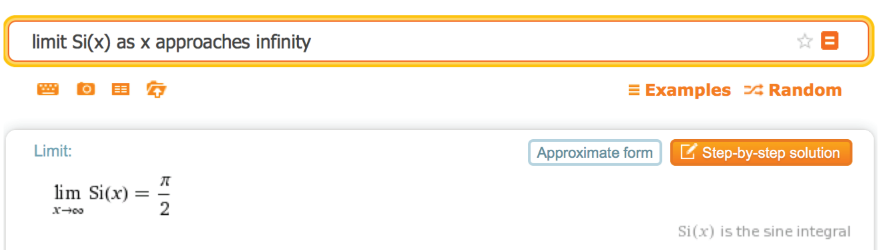
So the first inflection point to the right of the origin is $x \approx 4.4934$ if the sign of the second derivative changes. Using WolframAlpha or the TI-84, we compute $\text{Si}''(4.48) \approx -0.0131$ and $\text{Si}''(4.50) \approx 0.0064$, so the sign changes and there is an inflection point at $x \approx 4.4934$. The coordinates of that point are $(4.4934, \text{Si}(4.4934))$ or $(4.4934,\ 1.6556)$.

(d) *Does this function have horizontal asymptotes?*

We want to compute

$$\lim_{x\to\infty} \text{Si}(x) \qquad \text{and} \qquad \lim_{x\to-\infty} \text{Si}(x)$$

Using WolframAlpha,



and

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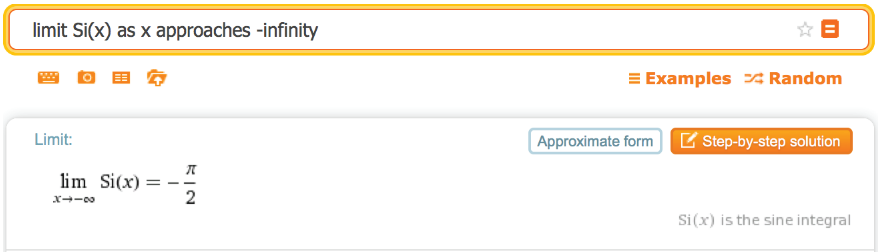

So the horizontal asymptotes for $\text{Si}(x)$ are $y = \pm\frac{\pi}{2}$.

(Note: I didn't know the Mathematica format to use with WolframAlpha, so I just typed a verbal description.)

(e) *Solve the following equation correct to one decimal place:*

$$\int_0^x \frac{\sin(t)}{t} \, \mathrm{d}t = 1$$

Using WolframAlpha,