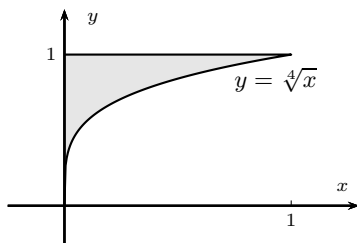


Calculus I, Section 5.4, #50
Indefinite Integrals and the Net Change Theorem

The boundaries of the shaded region are the y -axis, the line $y = 1$, and the curve $y = \sqrt[4]{x}$. Find the area of the region by writing x as a function of y and integrating with respect to y .¹



“...writing x as a function of y ” means we solve for x .

$$\begin{aligned}y &= \sqrt[4]{x} \\y^4 &= \sqrt[4]{x^4} \\y^4 &= x\end{aligned}$$

or

$$x = y^4$$

Now we can write and evaluate the definite integral.

$$\begin{aligned}\text{Area} &= \int_{y=0}^{y=1} y^4 \, dy \\&= \left[\frac{y^{4+1}}{4+1} \right]_{y=0}^{y=1} \\&= \left[\frac{y^5}{5} \right]_{y=0}^{y=1} \\&= \left(\frac{1^5}{5} \right) - \left(\frac{0^5}{5} \right) \\&= \frac{1}{5} - 0 \\&= \frac{1}{5}\end{aligned}$$

Thus the area of the shaded region is exactly $\frac{1}{5}$.

¹Stewart, *Calculus, Early Transcendentals*, p. 409, #50.