

Calculus I, Section 5.4, #66
Indefinite Integrals and the Net Change Theorem

Suppose that a volcano is erupting and readings of the rate $r(t)$ at which solid materials are spewed into the atmosphere are given in the table. The time t is measured in seconds and the units for $r(t)$ are tonnes (metric tons) per second.¹

t	0	1	2	3	4	5	6
$r(t)$	2	10	24	36	46	54	60

- (a) Give upper and lower estimates for the total quantity $Q(6)$ of erupted materials after 6 seconds.

Since the data in the table are rates in $\frac{\text{tonnes}}{\text{second}}$ and the input is in second, the definite integral will give us the net change in $\frac{\text{tonnes}}{\text{second}} \cdot \text{second} = \text{tonnes}$ for the first six seconds. Since our time data begins at $t = 0$, this net change will be the total quantity, assuming that the amount spewed into the atmosphere at $t = 0$ is 0 tonnes. (Note that this is NOT the number we are given in the table. The table tells us the rate, and it is reasonable to assume that the total amount at time $t = 0$ is 0 tonnes.)

$$Q(6) - Q(0) = \int_0^6 r(t) dt$$

and since $Q(0) = 0$,

$$Q(6) = \int_0^6 r(t) dt$$

The right endpoint estimate is

$$\begin{aligned} Q(6) &\approx (2 + 10 + 24 + 36 + 46 + 54) \cdot 1 \\ &= 172 \end{aligned}$$

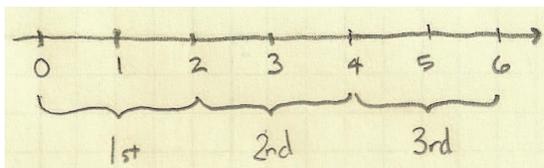
The left endpoint estimate is

$$\begin{aligned} Q(6) &\approx (10 + 24 + 36 + 46 + 54 + 60) \cdot 1 \\ &= 230 \end{aligned}$$

Thus the upper estimate for total quantity spewed in the first 6 seconds is 230 tonnes and the lower estimate is 172 tonnes.

- (b) Use the midpoint Rule to estimate $Q(6)$.

Because we are given data, we must choose our subintervals so that we know the value of $r(t)$ at the midpoint of the subinterval. From the drawing below, we can see that $\Delta t = 2$ and the midpoints of the three intervals are $t = 1, 3,$ and 5 .



Thus

$$\begin{aligned} Q(6) &\approx M_6 = (r(1) + r(3) + r(5)) \cdot 2 \\ &= (10 + 36 + 54) \cdot 2 \\ &= 200 \text{ tonnes} \end{aligned}$$

Thus the midpoint estimate for the total quantity spewed in the first 6 seconds is 200 tonnes.

¹Stewart, *Calculus, Early Transcendentals*, p. 410, #66.