Evaluate the indefinite integral.[1]

$$\int \sin(x)\sin(\cos(x))\ \mathrm{d}x$$

Let's closely examine the given integral:

For this integral, we have a cosine function inside a sine function, $\sin(\cos(x))$, and (almost) the derivative of the cosine as a factor, $\sin(x)$.

$$\int \underbrace{\sin(x)}_{\text{(almost) } \mathrm{d}u}\sin(\underbrace{\cos(x)}_{\text{inside}})\ \underbrace{\mathrm{d}x}_{\text{(almost) } \mathrm{d}u}$$

We let $u = \cos(x)$, so $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin(x)$ or $\mathrm{d}u = -\sin(x)\,\mathrm{d}x$.

To complete $\mathrm{d}u$ in the integral, we need a factor of -1. We will multiply inside the integral by -1, and outside the integral by -1.

$$\begin{aligned}
-1 \cdot \int -1 \cdot \sin(x)\sin(\cos(x))\ \mathrm{d}x \\
&= -1 \cdot \int \sin(\underbrace{\cos(x)}_{u}) \cdot \underbrace{-\sin(x)\ \mathrm{d}x}_{\mathrm{d}u} \\
&= -1 \cdot \int \sin(u)\ \mathrm{d}u \\
&= -1 \cdot [-\cos(u) + C] \\
&= \cos(u) - C
\end{aligned}$$

but $-C$ is just another constant, so we write

$$= \cos(u) + C$$

and rewriting in terms of the original variable x we get

$$= \cos(\cos(x)) + C$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 419, #22.