Evaluate the indefinite integral.[1]

$$\int x\sqrt{x+2}\,\mathrm{d}x$$

The given integral

$$\int x\sqrt{x+2}\,\mathrm{d}x$$

does not match the basic antiderviatives we know, and there is no algebra that we can do on the integrand.

There is only one choice for substitution.

$$\begin{aligned} &\text{Let} && u = x+2 \\ &\text{so} && \mathrm{d}u = 1\cdot\mathrm{d}x \\ &\text{or} && \mathrm{d}u = \mathrm{d}x \end{aligned}$$

We need to express all of the integrand in terms of the new variable u. There is still a factor of x in the integrand. Since $u = x+2$, we have $x = u-2$. Now we're ready to substitute

$$\begin{aligned} \int x\sqrt{x+2}\,\mathrm{d}x &= \int (u-2)\sqrt{u}\,\mathrm{d}u \\ &= \int u\cdot\sqrt{u} - 2\sqrt{u}\,\mathrm{d}u \\ &= \int u\cdot u^{1/2} - 2u^{1/2}\,\mathrm{d}u \\ &= \int u^{3/2} - 2u^{1/2}\,\mathrm{d}u \end{aligned}$$

Now the integral matches the basic antiderivatives we know. So

$$\begin{aligned} &= \frac{u^{3/2+1}}{\frac{3}{2}+1} - 2\cdot\frac{u^{1/2+1}}{\frac{1}{2}+1} + C \\ &= \frac{u^{5/2}}{\frac{5}{2}} - 2\cdot\frac{u^{3/2}}{\frac{3}{2}} + C \\ &= \frac{2}{5}\cdot u^{5/2} - 2\cdot\frac{2}{3}\cdot u^{3/2} + C \\ &= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C \end{aligned}$$

Now we substitute back to the original variable x

$$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

Thus

$$\int x\sqrt{x+2}\,\mathrm{d}x = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 419, #24.