Evaluate the definite integral.[1]

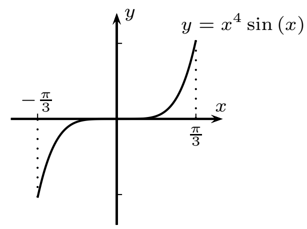
$$\int_{-\pi/3}^{\pi/3} x^4 \sin(x)\ \mathrm{d}x$$

Hmmmm ... the integral doesn't match any of our known antiderivatives ... and there doesn't seem to be any functions of other functions to do substitution with ...

Closely examine the integral again:

$$\int_{x=-\pi/3}^{x=\pi/3} x^4 \sin(x)\ \mathrm{d}x$$

The integrand has a factor of x^4, an even function of x, and a factor of $\sin(x)$, an odd function of x. Let's graph the integrand over the given interval of integration:



From the graph, it seems that the integrand is an odd function.[2] In addition, the interval of integration, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$, is symmetrical about the origin.

From the symmetry of the function and the interval, we know the area on the the left is equal to the area on the right. But the integral counts the area on the left as negative (the function is below the x-axis) and the area on the right as positive. These values add to zero, thus

$$\int_{x=-\pi/3}^{x=\pi/3} x^4 \sin(x)\ \mathrm{d}x = 0$$

Note:

a. If the interval was *not* symmetrical about the origin, we would *not* have been able to find a value for the integral.

b. We will learn how to find an antiderivative $\int x^4 \sin(x)\ \mathrm{d}x$ using *integration by parts* in a later lesson.

[1] Stewart, *Calculus, Early Transcendentals*, p. 419, #66.
[2] $f(x) = x^4 \sin(x)$ is an odd function because $f(-x) = (-x)^4 \sin(-x) = x^4 \cdot -\sin(x) = -x^4 \sin(x) = -f(x)$.