The growth rate of a fish population was modeled by the equation

$$G(t) = \frac{60{,}000\mathrm{e}^{-0.6t}}{(1+5\mathrm{e}^{-0.6t})^2}$$

where t is measured in years and G in kilograms per year. If the biomass was 25,000 kg *in the year 2000, what is the predicted biomass for the year 2020?*[1]

The units on the growth rate function are $\frac{\text{kg}}{\text{year}}$ and the units on t (and then Δt and $\mathrm{d}t$) are years, so the integral will give the net change in $\frac{\text{kg}}{\text{year}} \cdot \text{years} = \text{kg}$ from 2000 to 2020.

$$\text{net change from 2000 to 2020} = \int_{t=0}^{t=20} \frac{60{,}000\mathrm{e}^{-0.6t}}{(1+5\mathrm{e}^{-0.6t})^2}\,\mathrm{d}t$$

Let $u = 1 + 5\mathrm{e}^{-0.6t}$, so $\mathrm{d}u = 5 \cdot \mathrm{e}^{-0.6t} \cdot -0.6\mathrm{d}t$ or $\mathrm{d}u = -3\mathrm{e}^{-0.6t}\mathrm{d}t$.

Also when $t = 0$, $u = 6$ and when $t = 20$, $u = 1 + 5\mathrm{e}^{-0.6\cdot 20} = 1 + 5\mathrm{e}^{-12}$, so we get

$$\begin{aligned}
&= 60{,}000 \cdot \frac{1}{-3} \cdot \int_{u=6}^{u=1+5\mathrm{e}^{-12}} \frac{1}{u^2}\,\mathrm{d}u \\
&= -20{,}000 \int_{u=6}^{u=1+5\mathrm{e}^{-12}} u^{-2}\,\mathrm{d}u \\
&= -20{,}000 \left[\frac{u^{-1}}{-1}\right]_{u=6}^{u=1+5\mathrm{e}^{-12}} \\
&= -20{,}000 \cdot -1 \cdot \left[\frac{1}{u}\right]_{u=6}^{u=1+5\mathrm{e}^{-12}} \\
&= 20{,}000 \left[\frac{1}{1+5\mathrm{e}^{-12}} - \frac{1}{6}\right] \\
&\approx 16{,}666
\end{aligned}$$

This is the *change* in the biomass from 2000 to 2020.

Thus, the predicted biomass for 2020 is about $25{,}000 + 16{,}666 = 41{,}666$ kg.

[1] Stewart, *Calculus, Early Transcendentals*, p. 420, #84.