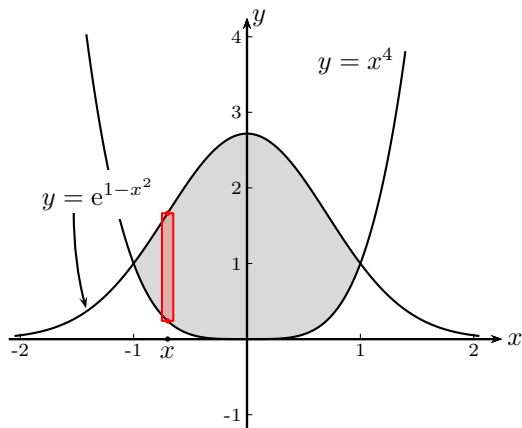


Calculus II, Section 6.1, #42
Areas Between Curves

Graph the region between the curves and use your calculator to compute the area correct to five decimal places.¹

$$y = e^{1-x^2}, \quad y = x^4$$

Let's sketch the graph so we have an idea of what we are working with.



The region of interest seems to start at $x = -1$ and to finish at $x = 1$, but we need to confirm this algebraically. We would usually solve $e^{1-x^2} = x^4$, but none of our algebraic techniques work.² However, if we substitute $x = -1$ into both equations, we get $y = 1$, and if we substitute $x = 1$ into both, we again get $y = 1$. This confirms algebraically what we observed on the graph. Also, using `calc:intersect` on the TI-84, we find the x -coordinate of the points of intersection to be $x \pm 1$.

The base of the representative rectangle is Δx , and the height is $(e^{1-x^2}) - (x^4)$, so the area of the representative rectangle is $((e^{1-x^2}) - (x^4)) \Delta x$. We generate representative rectangles from $x = -1$ to $x = 1$, so the area is given by

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n ((e^{1-x^2}) - (x^4)) \Delta x \\ &= \int_{x=-1}^{x=1} (e^{1-x^2}) - (x^4) \, dx \end{aligned}$$

If we split the integral

$$= \int_{x=-1}^{x=1} e^{1-x^2} \, dx - \int_{x=-1}^{x=1} x^4 \, dx$$

then the integral on the right is one of our basic forms, but the integral on the left is not. We do not know a basic antiderivative for e^{1-x^2} and if we try the obvious substitution $u = 1 - x^2$ so $du = -2x \, dx$, we do not have a factor of x to complete the substitution. At this point in our calculus knowledge, we're stuck, so we turn to technology to evaluate the definite integral.

¹Stewart, *Calculus, Early Transcendentals*, p. 435, #42.

²Give it a try!

Calculus II

Areas Between Curves

WolframAlpha gives the following result for this problem:

The screenshot shows the WolframAlpha interface. At the top, the input field contains the code `Integrate[e^(1-x^2)-x^4, {x,-1,1}]`. Below the input field are icons for keyboard, camera, list, and share. To the right are links for "Examples" and "Random". The main result area is titled "Definite integral:" and shows the equation $\int_{-1}^1 (e^{1-x^2} - x^4) dx = e\sqrt{\pi} \operatorname{erf}(1) - \frac{2}{5} \approx 3.66016$. A "More digits" button is located to the right of the equation. At the bottom right of the result area, it says "erf(x) is the error function".

Note that WolframAlpha gives us an exact value in terms of $\operatorname{erf}(1)$ and then tells us that this is called the “error function.” This is a function from advanced calculus. WolframAlpha then gives us the approximation $\int_{x=-1}^{x=1} e^{1-x^2} - x^4 dx \approx 3.66016$.

The TI-84 also includes a numerical approximation for definite integrals. We type

`fnInt(e^(1-x^2)-x^4,x,-1,1)`

press **ENTER**, and the calculator returns 3.660156939.

Thus, to five decimal places, the area of the region bounded by $y = e^{1-x^2}$ and $y = x^4$ is ≈ 3.66016 .