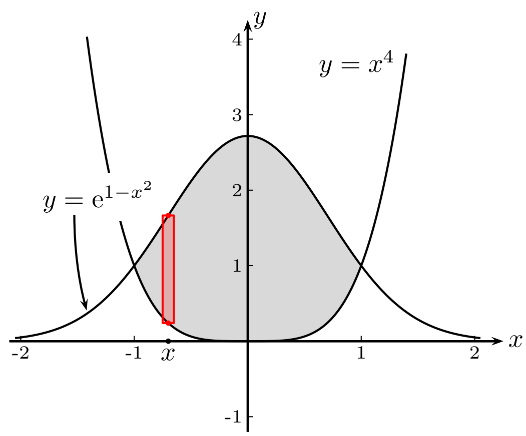
Graph the region between the curves and use your calculator to compute the area correct to five decimal places.[1]

$$y = e^{1-x^2}, \quad y = x^4$$

Let's sketch the graph so we have an idea of what we are working with.



The region of interest seems to start at $x = -1$ and to finish at $x = 1$, but we need to confirm this algebraically. We would usually solve $e^{1-x^2} = x^4$, but none of our algebraic techniques work.[2] However, if we substitute $x = -1$ into both equations, we get $y = 1$, and if we substitute $x = 1$ into both, we again get $y = 1$. This confirms algebraically what we observed on the graph. Also, using `calc:intersect` on the TI-84, we find the x-coordinate of the points of intersection to be $x \pm 1$.

The base of the representative rectangle is Δx, and the height is $\left(e^{1-x^2}\right) - \left(x^4\right)$, so the area of the representative rectangle is $\left(\left(e^{1-x^2}\right) - \left(x^4\right)\right)\Delta x$. We generate representative rectangles from $x = -1$ to $x = 1$, so the area is given by

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(e^{1-x^2}\right) - \left(x^4\right)\right)\Delta x$$
$$= \int_{x=-1}^{x=1} \left(e^{1-x^2}\right) - \left(x^4\right)\, dx$$

If we split the integral

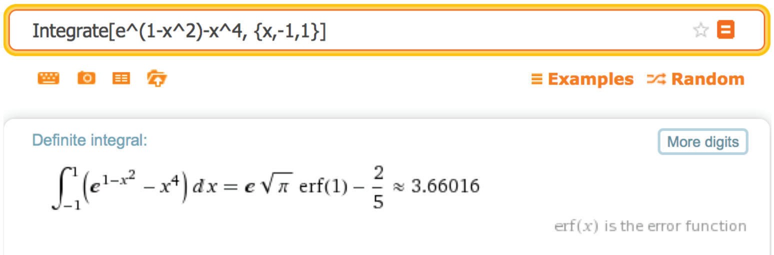
$$= \int_{x=-1}^{x=1} e^{1-x^2}\, dx - \int_{x=-1}^{x=1} x^4\, dx$$

then the integral on the right is one of our basic forms, but the integral on the left is not. We do not know a basic antiderivative for e^{1-x^2} and if we try the obvious substitution $u = 1 - x^2$ so $du = -2x\, dx$, we do not have a factor of x to complete the substitution. At this point in our calculus knowledge, we're stuck, so we turn to technology to evaluate the definite integral.

[1] Stewart, *Calculus, Early Transcendentals*, p. 435, #42.
[2] Give it a try!

Continued $\Longrightarrow$

WolframAlpha gives the following result for this problem:



Note that WolframAlpha gives us an exact value in terms of $erf(1)$ and then tells us that this is called the "error function." This is a function from advanced calculus. WolframAlpha then gives us the approximation $\int_{x=-1}^{x=1} \mathrm{e}^{1-x^2} - x^4 \, \mathrm{d}x \approx 3.66016$.

The TI-84 also includes a numerical approximation for definite integrals. We type

```
fnInt(e^(1-x^2)-x^4,x,-1,1)
```

press ENTER, and the calculator returns 3.660156939.

Thus, to five decimal places, the area of the region bounded by $y = \mathrm{e}^{1-x^2}$ and $y = x^4$ is ≈ 3.66016.