Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the given curves about the specified axis.[1]

$$y = \sqrt{x}, \quad x = 2y; \quad \textit{about } x = 5$$

In Figure 1, the curves are sketched along with the axis of revolution. Some simple algebra[2] confirms that the the curves intersect at (0,0) and (2,4). Since we are going to use cylindrical shells, the representative rectangle is parallel to the axis of revolution; this is sketched in Figure 2. The width of the representative rectangle is Δx, so everything in our integral will have to be in terms of x. The function $y = \sqrt{x}$ is already in terms of x, and some algebra transforms $x = 2y$ into $y = \frac{1}{2}x$.

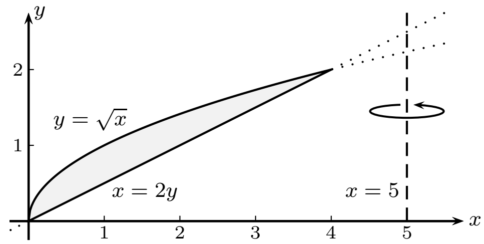

Figure 1: *The curves and the region.*

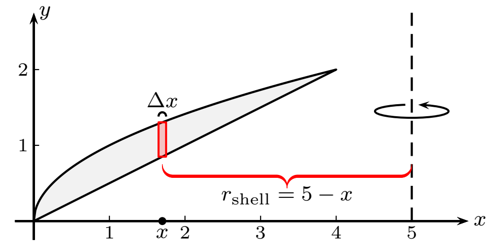

Figure 2: *The region with a representative rectangle.*

In Figure 3, we've sketched the shell generated by revolving the representative rectangle about $x = 5$. The shell, after it has been cut and flattened into a (very skinny) box, is sketched above. The volume of the shell is given by

$$V_{\text{shell}} = (\text{length of box})\,(\text{width of box})\,(\text{thickness of box})$$

and since the length of the box is the circumference of the shell

$$= (2\pi\,(5 - x))\left(\sqrt{x} - \frac{1}{2}x\right)(\Delta x)$$

We create shells from $x = 0$ to $x = 4$, so the volume, V, of the solid of revolution is given by

$$\begin{aligned} V &= \int_{x=0}^{x=4} 2\pi\,(5 - x)\left(\sqrt{x} - \frac{1}{2}x\right)\,\mathrm{d}x \\ &= 2\pi \int_{x=0}^{x=4} (5 - x)\left(x^{1/2} - \frac{1}{2}x\right)\,\mathrm{d}x \\ &= 2\pi \int_{x=0}^{x=4} 5x^{1/2} - \frac{5}{2}x - x^{3/2} + \frac{1}{2}x^2\,\mathrm{d}x \end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 454, #18.
[2] $y = \sqrt{x}$ and $x = 2y$, so $x = 2\sqrt{x}$ and $x^2 = 4x$. So $0 = 4x - x^2$ or $0 = x\,(4 - x)$ and thus $x = 0$ and $x = 4$.

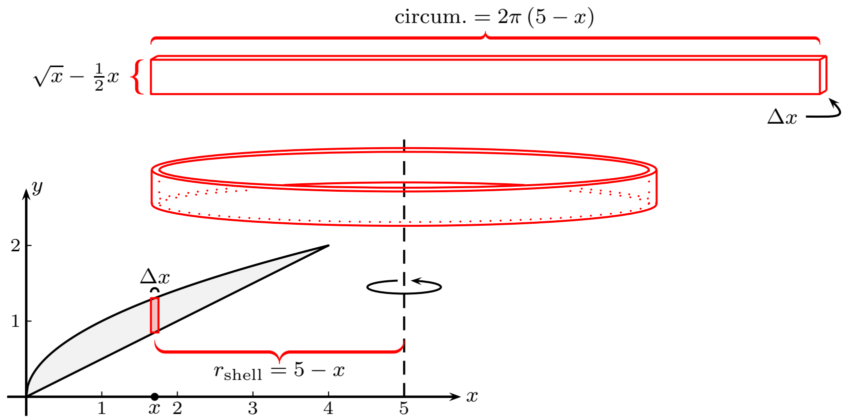

Figure 3: *The shell corresponding to the representative rectangle. The length of the flattened shell is not drawn to scale.*

$$
\begin{aligned}
V &= 2\pi \int_{x=0}^{x=4} 5x^{1/2} - \frac{5}{2}x - x^{3/2} + \frac{1}{2}x^2 \, \mathrm{d}x \\
&= 2\pi \left[5 \cdot \frac{x^{3/2}}{3/2} - \frac{5}{2} \cdot \frac{x^2}{2} - \frac{x^{5/2}}{5/2} + \frac{1}{2} \cdot \frac{x^3}{3} \right]_{x=0}^{x=4} \\
&= 2\pi \left[\frac{10x^{3/2}}{3} - \frac{5x^2}{4} - \frac{2x^{5/2}}{5} + \frac{x^3}{6} \right]_{x=0}^{x=4} \\
&= 2\pi \left[\left(\frac{10 \cdot 4^{3/2}}{3} - \frac{5 \cdot 4^2}{4} - \frac{2 \cdot 4^{5/2}}{5} + \frac{4^3}{6} \right) - \left(\frac{10 \cdot 0^{3/2}}{3} - \frac{5 \cdot 0^2}{4} - \frac{2 \cdot 0^{5/2}}{5} + \frac{0^3}{6} \right) \right] \\
&= 2\pi \left[\frac{80}{3} - 20 - \frac{64}{5} + \frac{64}{6} - 0 + 0 + 0 - 0 \right] \\
&= 2\pi \cdot \frac{68}{15} \\
&= \frac{136}{15}\pi
\end{aligned}
$$

Thus the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $x = 2y$ about $x = 5$ is $\frac{136}{15}\pi$ units^3.