*The Great Pyramid of King Khufu was built of limestone in Egypt over a 20-year period from 2580* BC *to 2560* BC*. Its base is a square with side length* 756 ft *and its height when built was* 481 ft*. (It was the tallest man-made structure in the world for more than 3800 years.) The density of limestone is about* $150\,\mathrm{lb/ft}^3$.[1]

(a) Estimate the total work done in building the pyramid.

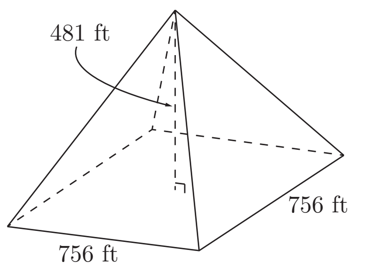


**Figure 1:** *The Great Pyramid*

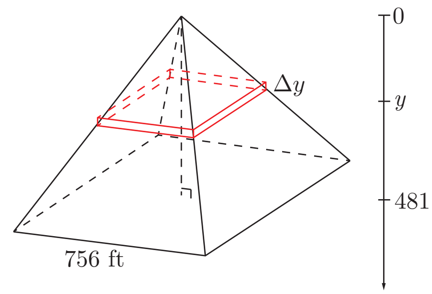


**Figure 2:** *A representative slice*

Figure 1 shows the Great Pyramid, with its height drawn perpendicular to its base. In Figure 2, we've drawn in a representative slice[2] $y$ ft from the top. We need to find the volume of this representative slice, determine the distance this representative slice has moved, and thus find the work done on the representative slice. We'll integrate to find the total work.

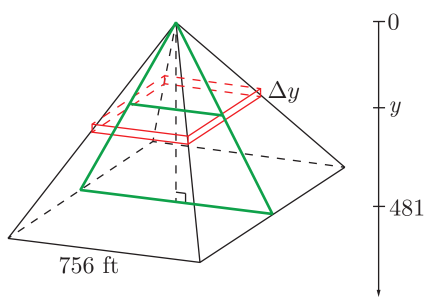


**Figure 3:** *A cross-section in 3-D.*

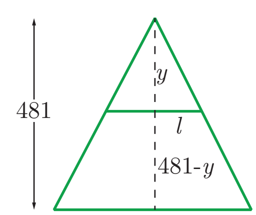


**Figure 4:** *A cross-section*

Figure 3 shows a cross section that we will use to find the length of the side of the representative slice. In Figure 4, we've drawn the cross-section and labeled the parts of the similar triangles. We need to find $l$, the length of the side of the representative slice. We have

$$\begin{aligned} \frac{481}{756} &= \frac{y}{l} \\ 481l &= 756y \\ l &= \frac{756}{481}y \end{aligned}$$

Then,

$$V_{\text{slice}} = \left(\frac{756}{481}y\right)^2 \Delta y$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 460, #34.

[2] For this problem, the base of the representative slice is actually a square.

We are given the density of limestone as $150\,\mathrm{lb/ft^3}$, so

$$\text{Force}_{\text{slice}} = 150 \cdot \frac{756^2}{481^2} y^2 \Delta y$$

As we build the pyramid, we are lifting blocks of limestone from the bottom to the current level. For our representative slice, this distance is $481 - y$. (See Figure 4.) So the work done on the slice is given by

$$\begin{aligned} W_{\text{slice}} &= 150 \cdot \frac{756^2}{481^2} y^2 \Delta y \cdot (481 - y) \\ &= 150 \cdot \frac{756^2}{481^2} y^2 (481 - y) \Delta y \end{aligned}$$

Finally, we create representative slices from $y = 0$ to $y = 481$, so the total work is

$$\begin{aligned} W &= \int_{y=0}^{y=481} 150 \cdot \frac{756^2}{481^2} y^2 (481 - y)\ \mathrm{d}y \\ &= 150 \cdot \frac{756^2}{481^2} \int_{y=0}^{y=481} y^2 (481 - y)\ \mathrm{d}y \\ &= 150 \cdot \frac{756^2}{481^2} \int_{y=0}^{y=481} 481y^2 - y^3\ \mathrm{d}y \\ &= 150 \cdot \frac{756^2}{481^2} \left[ 481 \cdot \frac{y^3}{3} - \frac{y^4}{4} \right]_{y=0}^{y=481} \\ &= 150 \cdot \frac{756^2}{481^2} \left[ \left( 481 \cdot \frac{481^3}{3} - \frac{481^4}{4} \right) - \left( 481 \cdot \frac{0^3}{3} - \frac{0^4}{4} \right) \right] \end{aligned}$$

The TI-84 gives us

$$\approx 1.65 \times 10^{12}$$

Thus, the total work done in building the Great Pyramid was $1.65 \times 10^{12}$ ft-lb.

(b) If each laborer worked for 10 hours a day for 20 years, for 340 days per year, and did 200 ft-lb/h of work in lifting limestone blocks into place, about how many laborers were needed to construct the pyramid?

After playing around with the units, we get

$$\left( \frac{1.65 \times 10^{12}\,\text{ft-lb}}{1\,\text{pyramid}} \right) \left( \frac{1\,\text{laborer}}{\frac{200\,\text{ft-lb}}{\text{h}}} \right) \left( \frac{1\,\text{day}}{10\,\text{h}} \right) \left( \frac{1\,\text{yr}}{340\,\text{day}} \right) \left( \frac{1\,\text{pyramid}}{20\,\text{yr}} \right)$$
$$\left( \frac{1.65 \times 10^{12}\,\text{ft-lb}}{1\,\text{pyramid}} \right) \left( \frac{1\,\text{laborer-h}}{200\,\text{ft-lb}} \right) \left( \frac{1\,\text{day}}{10\,\text{h}} \right) \left( \frac{1\,\text{yr}}{340\,\text{day}} \right) \left( \frac{1\,\text{pyramid}}{20\,\text{yr}} \right)$$
$$\approx 121{,}000\,\text{laborer}$$

Thus, about 121,000 laborers were needed to construct the pyramid.