Find the average value of the function on the given interval.[1]

$$f(u) = \frac{\ln(u)}{u}, \quad [1,5]$$

We know the average value of a continuous function on an interval $[a,b]$ is

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

so we compute

$$f_{\text{ave}} = \frac{1}{5-1} \int_1^5 \frac{\ln(u)}{u} \, du$$

We do not know a basic antiderivative for the integrand $\frac{\ln(u)}{u}$, so let's try substitution.

Let $w = \ln(u)$, so $dw = \frac{1}{u}du$. Also, when $u = 1$, $w = \ln(1) = 0$ and when $u = 5$, $w = \ln(5)$. We get

$$\begin{aligned}
f_{\text{ave}} &= \frac{1}{4} \int_{u=1}^{u=5} \ln(u) \cdot \frac{1}{u} \, du \\
&= \frac{1}{4} \int_{w=0}^{w=\ln(5)} w \, dw \\
&= \frac{1}{4} \left[\frac{w^2}{2} \right]_{w=0}^{w=\ln(5)} \\
&= \frac{1}{4} \cdot \left(\frac{(\ln(5))^2}{2} - \frac{0^2}{2} \right) \\
&= \frac{(\ln(5))^2}{8}
\end{aligned}$$

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[1]Stewart, *Calculus, Early Transcendentals*, p. 463, #8.