The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}\left(R^2 - r^2\right)$$

where P is the pressure differential between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect tor r) over the interval $0 \le r \le R$. Compare the average velocity to the maximum velocity.[1]

$$\begin{aligned}
v_{\text{ave}} &= \frac{1}{R-0}\int_{r=0}^{r=R} \frac{P}{4\eta l}\left(R^2 - r^2\right)\,\mathrm{d}r \\
&= \frac{1}{R}\cdot\frac{P}{4\eta l}\int_{r=0}^{r=R}\left(R^2 - r^2\right)\,\mathrm{d}r \\
&= \frac{P}{4\eta l R}\int_{r=0}^{r=R} R^2 - r^2\,\mathrm{d}r \\
&= \frac{P}{4\eta l R}\left[R^2 r - \frac{r^3}{3}\right]_{r=0}^{r=R} \\
&= \frac{P}{4\eta l R}\left[\left(R^2\cdot R - \frac{R^3}{3}\right) - \left(R^2\cdot 0 - \frac{0^3}{3}\right)\right] \\
&= \frac{P}{4\eta l R}\left(R^3 - \frac{R^3}{3}\right) \\
&= \frac{P}{4\eta l R}\left(\frac{2R^3}{3}\right) \\
&= \frac{PR^2}{6\eta l}
\end{aligned}$$

Thus the average velocity of blood in the blood vessel is $\frac{PR^2}{6\eta l}$.

Consider again the velocity function $v(r) = \frac{P}{4\eta l}\left(R^2 - r^2\right)$. Here, R is a constant and as r increases, the factor $R^2 - r^2$ decreases. Since all the other parameters—P, l, and η—are positive constants, the function $v(r)$ is a decreasing function. So the maximum value of the velocity occurs when $r = 0$, *i.e.*, the maximum velocity is

$$\begin{aligned}
v(0) &= \frac{P}{4\eta l}\left(R^2 - 0^2\right) \\
&= \frac{PR^2}{4\eta l}
\end{aligned}$$

and thus

$$v_{\text{ave}} = \frac{2}{3}v_{\text{max}}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 463, #18.