First make a substitution and then use integration by parts to evaluate the integral.[1]

$$\int_0^{\pi} \mathrm{e}^{\cos(t)} \sin(2t)\ \mathrm{d}t$$

The first thing we notice in the integrand is the factor $\sin(2t)$. This stands out because the argument[2] of the sine function is different than the argument for the cosine function. We'll start by using a well known trigonometric identity.

$$\begin{aligned}
&\int_{t=0}^{t=\pi} \mathrm{e}^{\cos(t)} \sin(2t)\ \mathrm{d}t \\
&\qquad = \int_{t=0}^{t=\pi} \mathrm{e}^{\cos(t)} \cdot 2\cos(t)\sin(t)\ \mathrm{d}t \\
&\qquad = 2 \cdot \int_{t=0}^{t=\pi} \mathrm{e}^{\cos(t)} \cos(t)\sin(t)\ \mathrm{d}t
\end{aligned}$$

Now we are ready to follow the instruction to "…make a substitution…". Let $x = \cos(t)$, so $\mathrm{d}x = -\sin(t)\ \mathrm{d}t$. Also, when $t = 0$, $x = 1$ and when $t = \pi$, $x = -1$.

$$\begin{aligned}
&= -1 \cdot 2 \cdot \int_{t=0}^{t=\pi} \mathrm{e}^{\cos(t)} \cos(t) \cdot -\sin(t)\ \mathrm{d}t \\
&= -2 \cdot \int_{x=1}^{x=-1} \mathrm{e}^{x} \cdot x \cdot\ \mathrm{d}x
\end{aligned}$$

(Sometimes students will worry about a smaller value for the uppr limit of integration than for the lower limit, and apply a property of definite integrals to switch the limits. This is not necessary. Not wrong, just not necessary.)

This new integral has an integrand that is a product, making it a great candidate for integration by parts.

Let $u = x$, so $\mathrm{d}v = \mathrm{e}^x\ \mathrm{d}x$, then $\mathrm{d}u = \mathrm{d}x$ and $v = \mathrm{e}^x$.[3] We have

$$\begin{aligned}
&= -2 \cdot \left[\left[x \cdot \mathrm{e}^x\right]_{x=1}^{x=-1} - \int_{x=1}^{x=-1} \mathrm{e}^x\ \mathrm{d}x \right] \\
&= -2 \cdot \left[\left(-1 \cdot \mathrm{e}^{-1}\right) - \left(1 \cdot \mathrm{e}^{1}\right) - \left[e^x\right]_{x=1}^{x=-1} \right] \\
&= -2 \cdot \left[-\frac{1}{\mathrm{e}} - \mathrm{e} - \left[\mathrm{e}^{-1} - \mathrm{e}^{1}\right] \right] \\
&= -2 \cdot \left[-\frac{1}{\mathrm{e}} - \mathrm{e} - \frac{1}{\mathrm{e}} + \mathrm{e} \right] \\
&= \frac{4}{\mathrm{e}}
\end{aligned}$$

Thus, $\displaystyle\int_0^{\pi} \mathrm{e}^{\cos(t)} \sin(2t)\ \mathrm{d}t = \frac{4}{\mathrm{e}}$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 477, #40.

[3] The input to a function is sometimes called the argument.

[3] The mnemonic LIATE indicates that we should choose an algebraic function (x) for u before we choose an exponential function (e^x).