Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves about the given axis.[1]

$$y = e^{-x}, \quad y = 0, \quad x = -1, \quad x = 0; \quad \textit{about } x = 1$$

In Figure 1, the curves are sketched along with the axis of revolution. Since we are going to use cylindrical shells, the representative rectangle is parallel to the axis of revolution; this is sketched in Figure 2. The width of the representative rectangle is Δx, so everything in our integral will have to be in terms of x. The function $y = e^{-x}$ is already in terms of x, so we are ready to proceed.

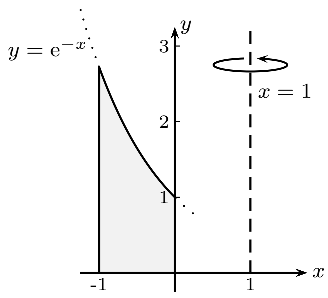

Figure 1: *The curves and the region.*

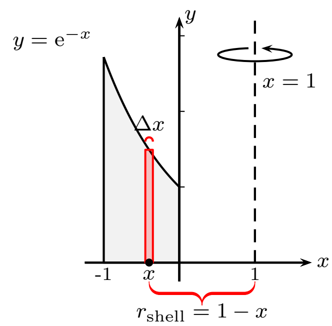

Figure 2: *The region with a representative rectangle.*

In Figure 3, we've sketched the shell generated when the representative rectangle is revolved about $x = 1$, along with the flattened shell. The volume of the shell is given by

$$V_{\text{shell}} = 2\pi\,(1 - x)\,e^{-x}\,\Delta x$$

We create shells from $x = -1$ to $x = 0$, so the volume, V, of the solid of revolution is given by

$$\begin{aligned} V &= \int_{x=-1}^{x=0} 2\pi\,(1-x)\,e^{-x}\,dx \\ &= 2\pi \int_{x=-1}^{x=0} (1-x)\,e^{-x}\,dx \end{aligned}$$

Referring to LIATE[2], we

Let	$u = 1 - x,$	$dv = e^{-x}\,dx$
so	$du = -\,dx,$	$v = -e^{-x}.$

We have

$$\begin{aligned} &= 2\pi \left[\left[(1-x)\,e^{-x}\right]_{x=-1}^{x=0} - \int_{x=-1}^{x=0} -e^{-x} \cdot -dx \right] \\ &= 2\pi \left[\left[(1-0)\left(e^{-0}\right) - (1-(-1))\left(-e^{-(-1)}\right)\right] - \int_{x=-1}^{x=0} e^{-x}\,dx \right] \\ &= \left[[-1 + 2e] - \left[-e^{-x}\right]_{x=-1}^{x=0} \right] \\ &= 2\pi \left[2e - 1 - \left[-e^{-0} - \left(-e^{-(-1)} \right) \right] \right] \\ &= 2\pi\,[2e - 1 - [-1 + e]] \\ &= 2\pi e \end{aligned}$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 477, #63.
[2]**L**ogarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigonometric, **E**xponential; a mnemonic for selecting the u-factor in inte-

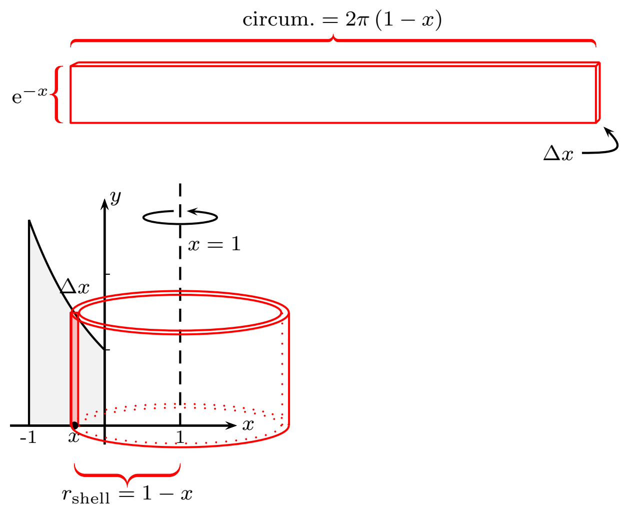

Figure 3: *The shell corresponding to the representative rectangle. Neither the height nor the length of the flattened shell are drawn to scale.*

Thus the volume of the solid obtained by rotating the region bounded by $y = \mathrm{e}^{-x}$, $y = 0$, $x = -1$, and $x = 0$ about $x = 1$ is $2\pi\mathrm{e}\,\text{units}^3$.

gration by parts.