Evaluate the integral.[1]

$$\int_{\pi/4}^{\pi/2} \csc^4(\theta)\cot^4(\theta)\ \mathrm{d}\theta$$

Cotangents and cosecants ... what do we know?

$$1+\cot^2(\theta)=\csc^2(\theta) \qquad \csc^2(\theta)-1=\cot^2(\theta)$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}[\cot(\theta)]=-\csc^2(\theta) \qquad \frac{\mathrm{d}}{\mathrm{d}\theta}[\csc(\theta)]=-\csc(\theta)\cot(\theta)$$

Let's rewrite the original integral

$$\begin{aligned}
&\int_{\theta=\pi/4}^{\theta=\pi/2} \cot^4(\theta)\csc^4(\theta)\ \mathrm{d}\theta \\
&\qquad=\int_{\theta=\pi/4}^{\theta=\pi/2} \cot^4(\theta)\csc^2(\theta)\csc^2(\theta)\ \mathrm{d}\theta \\
&\qquad=\int_{\theta=\pi/4}^{\theta=\pi/2} \cot^4(\theta)\left(1+\cot^2(\theta)\right)\csc^2(\theta)\ \mathrm{d}\theta \\
&\qquad=\int_{\theta=\pi/4}^{\theta=\pi/2} \left(\cot^4(\theta)+\cot^6(\theta)\right)\csc^2(\theta)\ \mathrm{d}\theta
\end{aligned}$$

and now we can do a basic u-substitution. Let $u=\cot(\theta)$, so $\mathrm{d}u=-\csc^2(\theta)\ \mathrm{d}\theta$. Also, when $\theta=\pi/4$, $u=\cot(\pi/4)=1$ and when $\theta=\pi/2$, $u=\cot(\pi/2)=0$. We have

$$\begin{aligned}
&=-1\cdot\int_{\theta=\pi/4}^{\theta=\pi/2} \left(\cot^4(\theta)+\cot^6(\theta)\right)\cdot-\csc^2(\theta)\ \mathrm{d}\theta \\
&=-1\cdot\int_{u=1}^{u=0} u^4+u^6\ \mathrm{d}u \\
&=-1\cdot\left[\frac{u^5}{5}+\frac{u^7}{7}\right]_{u=1}^{u=0} \\
&=-1\cdot\left[\left(\frac{0^5}{5}+\frac{0^7}{7}\right)-\left(\frac{1^5}{5}+\frac{1^7}{7}\right)\right] \\
&=-1\cdot\left[0+0-\frac{1}{5}-\frac{1}{7}\right] \\
&=-1\cdot-\frac{12}{35} \\
&=\frac{12}{35}
\end{aligned}$$

Thus,

$$\int_{\pi/4}^{\pi/2} \csc^4(\theta)\cot^4(\theta)\ \mathrm{d}\theta=\frac{12}{35}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 485, #38.