Household electricity is supplied in the form of alternating current that varies from $-155\,\mathrm{V}$ *to* $155\,\mathrm{V}$ *with a frequency of 60 cycles per second (Hz). the voltage is thus given by the equation*

$$E(t) = 155\sin(120\pi t)$$

where t *is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of* $[E(t)]^2$ *over on cycle.*[1]

(a) *Calculate the RMS voltage of household current.*

We need to compute the average value of $[E(t)]^2$ over on cycle, *i.e.*, from $t = 0\,\mathrm{s}$ to $t = \frac{1}{60}\,\mathrm{s}$ since there are 60 cycles per second. So

$$\begin{aligned}
[E(t)]^2_{\text{average}} &= \frac{1}{\frac{1}{60} - 0} \int_{t=0}^{t=1/60} (155\sin(120\pi t))^2 \,\mathrm{d}t \\
&= 60 \cdot \int_{t=0}^{t=1/60} 155^2 \sin^2(120\pi t)\,\mathrm{d}t \\
&= 60 \cdot 155^2 \cdot \int_{t=0}^{t=1/60} \sin^2(120\pi t)\,\mathrm{d}t
\end{aligned}$$

We will use the half-angle formula, $\sin^2(x) = \frac{1-\cos(2x)}{2}$ to integrate $\sin^2(120\pi t)$, so let's do a substitution to transform $\sin^2(120\pi t)$ to a simpler form.

Let $u = 120\pi t$, so $\mathrm{d}u = 120\pi \mathrm{d}t$. Also, when $t = 0$, $u = 0$, and when $t = 1/60$, $u = 2\pi$. We have

$$\begin{aligned}
&= 60 \cdot 155^2 \cdot \frac{1}{120\pi} \int_{t=0}^{t=1/60} \sin^2(120\pi t) \cdot \frac{1}{120\pi} \cdot \mathrm{d}t \\
&= \frac{155^2}{2\pi} \int_{u=0}^{u=2\pi} \sin^2(u)\,\mathrm{d}u \\
&= \frac{155^2}{2\pi} \int_{u=0}^{u=2\pi} \frac{1-\cos(2u)}{2}\,\mathrm{d}u \\
&= \frac{155^2}{2\pi} \int_{u=0}^{u=2\pi} \frac{1}{2} - \frac{\cos(2u)}{2}\,\mathrm{d}u \\
&= \frac{155^2}{2\pi} \left[\int_{u=0}^{u=2\pi} \frac{1}{2}\,\mathrm{d}u - \int_{u=0}^{u=2\pi} \frac{\cos(2u)}{2}\,\mathrm{d}u \right] \\
&= \frac{155^2}{2\pi} \left[\int_{u=0}^{u=2\pi} \frac{1}{2}\,\mathrm{d}u - \frac{1}{2} \int_{u=0}^{u=2\pi} \cos(2u)\,\mathrm{d}u \right] \\
&= \frac{155^2}{2\pi} \left[\left[\frac{1}{2}u \right]_{u=0}^{u=2\pi} - \frac{1}{2} \int_{u=0}^{u=2\pi} \cos(2u)\,\mathrm{d}u \right] \\
&= \frac{155^2}{2\pi} \left[\left(\frac{1}{2} \cdot 2\pi - \frac{1}{2} \cdot 0 \right) - \frac{1}{2} \left[\frac{1}{2}\sin(2u) \right]_{u=0}^{u=2\pi} \right] \\
&= \frac{155^2}{2\pi} \left[\pi - \frac{1}{2} \left[\left(\frac{1}{2}\sin(2 \cdot 2\pi) \right) - \left(\frac{1}{2}\sin(2 \cdot 0) \right) \right] \right] \\
&= \frac{155^2}{2\pi} \left[\pi - \frac{1}{2} [0 - 0] \right]
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 485, #66.

$$= \frac{155^2}{2\pi}[\pi]$$
$$= \frac{155^2}{2}$$

Thus the RMS of household current is $\sqrt{\frac{155^2}{2}} \approx 110\,\text{V}$.

(b) *Many electric stoves require an RMS voltage of* $220\,\text{V}$*. Find the corresponding amplitude* A *needed for the voltage* $E(t) = A\sin(120\pi t)$*.*

Here we need to solve

$$220 = \sqrt{\left[E(t)^2_{\text{average}}\right]}$$

or

$$\begin{aligned}
220^2 &= \frac{1}{1/60}\int_{t=0}^{t=1/60} A^2\sin^2(120\pi t)\ \mathrm{d}t \\
220^2 &= 60A^2\int_{t=0}^{t=1/60} \sin^2(120\pi t)\ \mathrm{d}t \\
220^2 &= 60A^2\int_{t=0}^{t=1/60} \frac{1}{2} - \frac{1}{2}\cos(240\pi t)\ \mathrm{d}t \\
220^2 &= 60A^2\cdot\frac{1}{2}\left[t - \frac{1}{240\pi}\sin(240\pi t)\ \mathrm{d}t\right]_{t=0}^{t=1/60} \\
220^2 &= 30A^2\left[\left(\frac{1}{60} - \frac{1}{240\pi}\sin\left(240\pi\cdot\frac{1}{60}\right)\right) - \left(0 - \frac{1}{240\pi}\sin(240\pi\cdot 0)\right)\right] \\
220^2 &= 30A^2\left[\frac{1}{60} - 0 - 0 + 0\right] \\
220^2 &= \frac{1}{2}A^2 \\
2\cdot 220^2 &= A^2
\end{aligned}$$

And, since A is a positive number, we get

$$\sqrt{2\cdot 220^2} = A$$

or

$$311 \approx A$$

Thus the needed amplitude is about $311\,\text{V}$.